Non-Fermi liquid phenomena at the quantum critical point

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1. Correlated metals: phenomenology

- Fermi-liquid theory
  - Strongly renormalised electron gas
  - Quasiparticles with large effective mass
  - Narrow band of conduction electrons

\[
\frac{m^*}{m_e} = \frac{v_F}{v_F^*} = \frac{T_F}{T_F^*} \approx 100
\]

- Thermal, magnetic and transport properties follow Fermi-liquid expressions \((T \rightarrow 0)\)
  - Specific heat
  - Electrical resistivity
  - Magnetic susceptibility

\[
c = \gamma^* T \quad \text{with} \quad \gamma^* / \gamma = m^* / m
\]

\[
\rho = \rho_0 + A^* T^2
\]

\[
\chi^* = \text{constant}
\]

- One-parameter scaling

\[
\frac{\chi^*}{\gamma^*} = 1
\]

\[
A^* \frac{1}{(\gamma^*)^2} \text{ is constant}
\]
Correlated metals: microscopic level

- Spin fluctuations of 4f (cerium) or 5f (uranium) electrons
- In dense Kondo system competing interactions:
  - On-site Kondo effect
  - Inter-site RKKY interaction
- Around critical value $J_c$ heavy-fermion state
  - $J < J_c$ magnetic order
  - $J > J_c$ Fermi liquid
- At $J_c$ quantum critical point
Correlated metals: renormalisation group picture

- Local-$f$-moments at high $T$ evolve into narrow hybridisation-induced band of quasiparticles at low $T$
- Ratio $T_K/T_{RKKY}$ determines groundstate
- AFM and FL fixed points connected by unstable non-Fermi liquid fixed point

Because of strong hybridisation correlated metals are excellent materials to control $T_K/T_{RKKY}$ by mechanical or chemical pressure at $T=0$

$\rightarrow$ quantum phase transition

Schematic RG flow diagram: Coleman, Physica B 1999
Quantum phase transition

- Continuous phase transition (second order) at $T=0$
  $\rightarrow$ quantum effects (energy scale $\hbar \omega_c$) are important
  $\hbar \omega_c \gg k_B T$

- Characteristic energy scale for quantum fluctuations vanishes as
  $\hbar \omega_c \sim |\Delta|^z$ with $\Delta = (1 - \delta/\delta_c)$

- Length scale $\xi$ diverges as $\xi^{-1} \sim |\Delta|^\nu$ and $\hbar \omega_c \sim \xi^{-z}$

- Static and dynamical critical behaviour coupled through dynamic critical exponent $z$ and correlation length exponent $\nu$
Non-Fermi liquid behaviour

- Effect of non-zero temperature on quantum critical points in itinerant Fermion systems (Millis PRB 1993; Hertz PRB 1976)
- Three regimes
  I Quantum Fermi liquid: fluctuations on scale of $\xi$ have energy $\gg k_B T$
  II Perturbative classical: $\xi$ controlled by $\delta$
    $$\xi^{-2} \sim |\delta - \delta_c|$$
  III Classical Gaussian: $\xi$ controlled by $T$
    $$\xi^{-2} \sim T^{1+1/z}$$

Schematic phase diagram: Millis, PRB 1993
NFL expressions for specific heat and resistivity

- Non-Fermi-liquid regime (II and III)
  - $z=3$: ferromagnetic quantum critical point
    \[ c \sim -T \ln(T/T_0) \quad \rho \sim T^{5/3} \]
    \[ c \sim T^{2/3} \quad \rho \sim T^{4/3} \]
  - $z=2$: antiferromagnetic quantum critical point
    \[ c \sim \gamma T - \alpha T^{3/2} \quad \rho \sim T^{3/2} \]
    \[ c \sim -T \ln(T/T_0) \quad \rho \sim T \]

- Fermi-liquid regime (I); $z=2$ or 3, $d=3$
  \[ c = \gamma T + \alpha T^3 \ln(T/T_0) \quad \rho = \rho_0 + AT^2 \]
2. Magnetotransport model of Rosch

- Weak-coupling spin-density wave model for nearly AF Fermi liquid
  - Heavy quasiparticles scatter at spin fluctuations
  - Dominant scattering near hot lines, i.e. points on the Fermi surface connected by AF wave vector $Q$

- Disorder taken into account
  - Boltzmann equation treatment of the interplay of strongly anisotropic scattering and weak isotropic impurity scattering

Rosch, PRB 2000
Resistivity scaling diagram

- Scaling form:
  \[ \Delta \rho = \rho_0 + \Delta \rho \]

  \[ \Delta \rho = T^{3/2} f \left( \frac{T}{\rho_0}, \frac{\delta - \delta_c}{\rho_0}, \frac{B}{\rho_0^{3/2}} \right) \]

- Three regimes:
  1. Disorder dominated NFL
     \[ \Delta \rho \sim t^{3/2} \]
  2. Clean limit NFL
     \[ \Delta \rho \sim t^{1/2} \]
  3. Fermi-liquid
     \[ \Delta \rho \sim t^2 r^{-1/2} \]

  where \( t = T/T_{coh}, r = (\delta - \delta_c)/\delta_c \) and \( x = \rho_0 / \rho_m \sim 1/RRR \)

Rosch, PRB 2000
3. Case study: $U_2Pt_2In$

- Member of tetragonal $U_2T_2X$ family with $T$ = transition metal, $X$ = In or Sn
  - 5f hybridisation phenomena
  - Doniach diagram
- Non-ordering heavy-fermion system
  - $\alpha T = 0.41$ J/mol\(_0\)K\(^2\) at $T=1$ K
- Non-Fermi liquid behaviour
  - $\alpha T \sim \ln(T/T_0)$
  - $\chi(T)$ has maximum at $T_{max} = 8$ K for $B||c$
  - $\chi(T) \sim T^{0.7}$ for $B||a$

Specific heat: Estrela et al., Physica B 1999
Case study: U$_2$Pt$_2$In

- Non-Fermi liquid behaviour in resistivity
  
  \[ \rho = \rho_0 + a T^\alpha \]

  - \( \alpha = 1.25 \pm 0.05 \) for \( l \parallel a \) (\( T < 1K \))
  - \( \alpha = 0.9 \pm 0.1 \) for \( l \parallel c \) (\( T \rightarrow 0 \))

- Fermi liquid \( \alpha = 2 \) recovered in magnetic field

- Kondo disorder?
  - \( \rho_0 = 115 \mu\Omega\text{cm} \) for \( l \parallel a \)
  - \( \rho_0 = 210 \mu\Omega\text{cm} \) for \( l \parallel c \)
  
  But

  - stoichiometric compound
  - high-crystalline quality from x-rays and neutrons

- Resistivity in field: Estrela et al., cond-mat/0009324
Stability of NFL phase to pressure

- Resistivity under pressure shows strong current-direction dependence
  - $I || a$
    - $\rho_a(T)$ reduced
    - recovery $T^2$ term near 1 GPa
  - $I || c$
    - $\rho_c(T)$ increases
    - minimum develops near 1 GPa
      - $T_{\text{min}} \sim 4.8 \text{ K at } 1.8 \text{ GPa and } B=0$
      - $T_{\text{min}} \sim 2.2 \text{ K at } 1.8 \text{ GPa and } B=8 \text{ T}$

Resistivity under pressure: Estrela et al., cond-mat/0009324
Stability of NFL phase to pressure

- For $l||a$ and $T \rightarrow 0$

$$\alpha = \lim_{T \rightarrow 0} \alpha_{\text{eff}}(T)$$

$$\alpha_{\text{eff}}(T) = 1 + \frac{d \ln (\frac{d \rho}{d T})}{d \ln T}$$

- $\alpha = 1.25 \pm 0.05$ at $p = 0$
- $\alpha = 2.0 \pm 0.1$ at $p \geq 1 \text{ GPa}$

- Coefficient of the $T^2$ term
  - $A = 2.1 \pm 0.2 \mu \Omega \text{cm/K}^2$ at $p = 1.0 \text{ GPa}$
  - $A = 0.4 \pm 0.04 \mu \Omega \text{cm/K}^2$ at $p = 1.8 \text{ GPa}$

- Recovery of Fermi-liquid behaviour near 1 GPa

Resistivity exponent $\alpha$ under pressure:

Estrela et al., cond-mat/0009324

WZI Group meeting "Condensed Matter Physics & Materials Science", Amsterdam, 21 March 2001
Analysis in magnetotransport model of Rosch

- For $I \parallel a$, $x \sim 1/\text{RRR} \sim 0.6$
  - intermediate disorder
- Fit $\rho \sim T^2$ and $\rho \sim T$
  at each pressure

Resistivity at $\rho = 1.8$ GPa: Estrela et al., cond-mat/0009324
Analysis in magnetotransport model of Rosch

From fit $\rho \sim T^2$ and $\rho \sim T$:

- $\Delta\rho \sim t^2 r^{-1/2}$
  - $T_{FL} = \rho - \rho_c$ with $\rho_c = 0$

- $\Delta\rho \sim t^x r^{1/2}$
  when $x < T/T_{coh} < x^{1/2}$
  - $\rho \sim T$ for $2.8 \text{ K} < T < 4.7 \text{ K}$
  - $x = 0.34$ and $T_{coh} = 8.1 \text{ K}$

- $\Delta\rho \sim t^\alpha$ with $1 < \alpha < 2$
  - represents $\Delta\rho \sim t^{3/2}$
  - $\alpha = 1.25 \pm 0.05$ at $p = 0$

For $l \parallel a$ the data are consistent with an AF QCP at $p_c = 0$
Doniach-type diagram for $U_2Pt_2In$ and $U_2Pd_2In$

- $U_2Pd_2In$
  - AF ordering at $T_N = 37 \text{ K}$
  - isoelectronic to $U_2Pt_2In$

- Doniach-type diagram
  - $J \propto V_{\sigma f}^{-1}/(E_F - E_f)$
  - $V_{\sigma} = (V_{df}^{-2} + V_{pf}^{-2} + V_{ff}^{-2})^{1/2}$

Doniach-type diagram: Estrela, Ph.D. Thesis 2000
Case study: U$_3$Ni$_3$Sn$_4$

- Member of cubic U$_3$T$_3$Sn$_4$ family where T= Ni, Cu, Pt, Au
  - 5f hybridisation phenomena
- Non-ordering heavy-fermion system
  - $\gamma = 0.128$ J/mol K$^2$
  - $c \sim \gamma T + T^3 \ln(T/T_0)$
  - Fermi liquid $T < 0.4$ K
- NFL for $0.3$ K $< T < 5$ K
  - $c \sim \gamma T - \alpha T^{3/2}$ (AF QCP)
  - $\chi \sim T^{-0.3}$ for $T=1.7-10$ K

Specific heat: Shlyk et al., J. Phys.: Cond. Mat. 1999
Stability of FL phase to pressure

- Resistivity typical for dense Kondo system
  - weak maximum at \( \approx 240 \text{ K} \)
  - \( T_{\text{coh}} \approx 16 \text{ K} \) at \( p=0 \)
  - \( T_{\text{coh}} \approx 20 \text{ K} \) at \( p=1.8 \text{ GPa} \)
  - \( \rho_0 \approx 6 \mu\Omega\text{cm}, \text{ RRR} \approx 55 \)
  - relatively clean material

Resistivity under pressure: Estrela et al., cond-mat/0009324
Stability of FL phase to pressure

- $\rho = \rho_0 + AT^2$ for $T < T_{FL}$
- Fermi-liquid temperature range increases with pressure

Resistivity under pressure versus $T^2$
Estrela et al., cond-mat/0011061
Analysis within magnetotransport model of Rosch

$\text{U}_3\text{Ni}_3\text{Sn}_4$ is relatively clean

- $\Delta \rho \sim t^2 r^{-1/2}$
- $\Delta \rho \sim t^{3/2}$ is suppressed
  
  $T_{FL} = a(p - p_c)^\nu$

  with $\nu = 0.50 \pm 0.07$ and
  $p_c = -0.04 \pm 0.02$ GPa

- $\Delta \rho \sim t x^{1/2}$
  
  when $T_{FL} < T < T_{coh} x^{1/2}$
  
  - $x = 0.018$ and $T_{coh} \sim 16$ K
  - $\rho \sim T$ predicted for
    $0.4$ K $< T < 2.1$ K

AF QCP at $p_c = -0.04$ GPa

$T_{FL}$ versus pressure: Estrela et al., cond-mat/0011061
Summary

- New magnetotransport model of Rosch for a nearly AF Fermi liquid
  - weak-coupling AF spin-density wave scenario
  - delineate different FL and NFL phases by magnetotransport experiments
  - includes the effects of disorder on the quantum phase transition

- Transport experiments under pressure to investigate magnetic QCP in two correlated metals
  - $U_2Pt_2In \rightarrow$ case of intermediate disorder
    \[ T_{FL} \sim p-p_c, \text{ AF QCP at } p_c = 0 \]
  - $U_3Ni_3Sn_4 \rightarrow$ case of relatively clean material
    \[ T_{FL} \sim (p-p_c)^{1/2}, \text{ AF QCP at } p_c = -0.04 \text{ GPa} \]
Outlook

• $U_2Pt_2In$
  - Why strong current-direction dependent pressure effect?
  - How to reconcile the $\sim T \ln(T/T_0)$ with an AF QCP? Two-dimensional nature of fluctuations like in CeCu$_{5.9}$Au$_{0.1}$?
  - Can one make $U_2Pt_2In$ magnetic by alloying with e.g. Th or Pd?

• $U_3Ni_3Sn_4$
  - Can one recover $\Delta \rho \sim T^{3/2}$ term by reducing $\rho_0$?
  - Can one make $U_3Ni_3Sn_4$ magnetic by alloying with e.g. Th or Pt?

• Transport in a magnetic field

• Weak-coupling versus strong-coupling scenario for the magnetic quantum phase transition in correlated metals
  Coleman, Physica B 1999; Schröder et al., Nature 2000