The QHE as laboratory system to study quantum phase transitions

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- Quantum Hall effect
- Critical behavior and scaling
- Magnetotransport at the plateau-insulator transition
- Scaling functions and RG flow diagram
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Further reading:
• Pruisken et al., cond-mat/0109043
• De Lang, Ph.D. thesis, 30 June 2005
The quantum Hall effect

- Magnetotransport 2DEG

\[ R_{xy} = \rho_{xy} = \frac{1}{i e^2} \frac{h}{e^2} \]

InGaAs/AlGaAs

\[ n = 2.8 \times 10^{15} \text{ m}^{-2} \]

\[ \mu = 3.4 \text{ m}^2/\text{Vs} \]

\[ T = 30 \text{ mK} \]
Simple “explanation” QHE

- Landau quantization

\[ E_n = (n + \frac{1}{2}) \frac{\hbar e}{m^*} B \]

- Disorder \( \rightarrow \) metal-insulator transitions at \( \nu = n + 1/2 \)

filling fraction \( \nu = \frac{h n_e}{e B} \)
Critical behavior & length scales

\[ \xi \propto \left| B-B_c \right|^{-\chi} \]

Localization length

Sample length \( L \to \phi \)

\[ L_\phi \propto T^{-p/2} \]

\[ \xi \geq L_\phi \quad \text{metal} \]

\[ \xi \ll L_\phi \quad \text{insulator} \]
Scaling of conductivity

\[
\sigma_{ij}(T, B) = g_{ij}(\left[\frac{L_\varphi}{\xi}\right]^{1/\chi}) \\
= g_{ij}(T^{-p/2\chi}(B - B_c)) \\
= \sigma_{ij}(X)
\]

scaling variable

\[
X = \frac{\Delta \nu}{\nu_0(T)}
\]

\[
\Delta \nu = \nu - \nu_c
\]

\[
\nu_0(T) = \left(\frac{T}{T_0}\right)^\kappa
\]

\[
\kappa = \frac{p}{2\chi}
\]

Pruisken, PRL 1988
The $\sigma_{xx} - \sigma_{xy}$ plane

\[ \sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \]

\[ \sigma_{xy} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2} \]
RG flow diagram

Pruisken, PRL 1988

Transitions at unstable fixed points

\[
\sigma_{xx,c} = \frac{1}{2}
\]

\[
\sigma_{xy,c} = n + \frac{1}{2}
\]

Scaling fields

\[
\sigma = \sigma_{xx} - \frac{1}{2}
\]

\[
\theta = \sigma_{xy} - n - \frac{1}{2}
\]
Magnetotransport at the plateau-insulator transition

• At PI transition $R_H$ quantization
  → separate scaling behavior from effects of sample imperfections

• Transport tensor with isotropic $\rho_0, \rho_H$

\[ \rho_{ij}(B) = S_{ij}\rho_0(B) + \epsilon_{ij}\rho_H(B) \]

- variations of carrier density
- contact misalignment
Experimental results

- Plateau-insulator transition at $B_c = 17.2$ T
- Quantized Hall insulator $T \leq 1.2$ K

$$\rho_H(B) = \frac{1}{2} (\rho_{xy}(B^\uparrow) + \rho_{xy}(B^\downarrow))$$

$$\rho_{xx}(B^\uparrow) = \rho_{xx}(B^\downarrow)$$

de Lang et al., Physica E 2002

InGaAs/InP heterostructure

$B$ up

$B$ down

$n = 2.2 \times 10^{15}$ m$^{-2}$  
$\mu = 1.6$ m$^2$/Vs
Longitudinal resistivity $\rho_0$

- Scaling yields $\kappa$

$$\rho_0(X) = \rho_c e^{-X - O(X^3)}$$

$$X = \Delta \nu / \nu_0(T)$$

$$\nu_0(T) = (T / T_0)^\kappa$$

$\kappa = 0.57 \pm 0.02$

$T_0 = 188 \pm 20$ K

$\nu_c = 0.55$

$v - \nu_c \sim \frac{1}{B} - \frac{1}{B_c}$

$T = 4.2$ K

$T = 0.16$ K

Scaling yields $\kappa$
Hall resistivity $\rho_H$

- Corrections to scaling $\rightarrow 2^{nd}$ critical exponent $\eta_\sigma$

$$\rho_H(X) = 1 + \eta(T)\rho_0(X)$$

$$\eta(T) = \left(\frac{T}{T_1}\right)^{\eta_\sigma}$$

$\sigma_\eta = 2.5 \pm 0.1$

$T_1 = 9.8 \pm 0.3K$

$\chi = 0$
Renormalization Group flow diagram

\[ T = 0.01 \text{ K} \]

\[ T = 10 \text{ K} \]

\[ \kappa = 0.57 \]
\[ y_\sigma = 2.5 \]
\[ T_0 = 188 \text{ K} \]
\[ T_1 = 9.8 \text{ K} \]

- Universal scaling functions

\[ \sigma_0 = \frac{\rho_0}{\rho_0^2 + 1 + 2\eta\rho_0} \]
\[ \sigma_H = \frac{1 + \eta\rho_0}{\rho_0^2 + 1 + 2\eta\rho_0} \]

- Beta functions

\[ \frac{d\sigma_{xx}}{d \ln L} = \beta_{xx}(\sigma_{xx}, \sigma_{xy}) \]
\[ \frac{d\sigma_{xy}}{d \ln L} = \beta_{xy}(\sigma_{xx}, \sigma_{xy}) \]
Comparing PI and PP

- Particle hole symmetry for PI transition only

\[
\sigma_0(X) = \sigma_0(-X) \\
\sigma_H(X) = 1 - \sigma_H(-X)
\]
Summary

• QHE is a quantum critical phenomenon

• PI transition: disentwine macroscopic sample inhomogeneities and intrinsic universal aspects

• Magnetotransport at PI transition
  - $\rho_0$ proper scaling
  - $\rho_H$ quantized, deviations $\rightarrow$ corrections to scaling

• Flow lines $\sigma_0(\sigma_H)$ at the PI transition have been extracted from the experiment
  $\rightarrow$ universal scaling functions