Available online at www.sciencedirect.com
SCIENCE DIRECT®
Solid State Communications 137 (2006) 540–544
solid state communications
www.elsevier.com/locate/ssc

Universal scaling results for the plateau–insulator transition in the quantum Hall regime

A.M.M. Pruisken [a], D.T.N. de Lang [b], L.A. Ponomarenko [b], A. de Visser [b,*]

[a] Institute for Theoretical Physics, University of Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands
[b] Van der Waals–Zeeman Institute, University of Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands

Received 15 June 2005; received in revised form 22 December 2005; accepted 12 January 2006 by M. Heiblum
Available online 31 January 2006

Abstract

The plateau–insulator (PI) transition in the quantum Hall regime, in remarkable contrast to the plateau–plateau (PP) transition, exhibits very special features that enable one for the first time to disentwine the quantum critical aspects of the electron gas (scaling functions, critical indices) from the sample dependent effects of macroscopic inhomogeneities (contact misalignments, density gradients). In this communication we report new experimental data taken from the PI transition of a low-mobility InGaAs/InP heterostructure and propose universal scaling functions for the transport coefficients. Our new findings elucidate fundamental theoretical aspects of quantum criticality that have so far remained inaccessible.

PACS: 73.43.−f; 73.43.Cd; 73.43.Nq

Keywords: D. Quantum Hall effect; D. Plateau–insulator transition; D. Universal scaling functions

1. Introduction

Following the pioneering experiments on quantum criticality in the quantum Hall regime by Wei et al. [1], the primary focus of subsequent work [2] has been on the critical exponent $\kappa \approx 0.42$ that characterizes the width $\nu_0(T) \propto T^{\kappa}$ of the plateau transitions as the temperature T approaches absolute zero [3]. However, even currently the exact meaning of κ and the very nature of the quantum Hall plateau transition remain a major topic of research. For example, one of the main difficulties in probing quantum criticality in the quantum Hall regime at finite T is that the experiment must be performed on samples where the dominant scattering mechanism is provided by short-ranged potential fluctuations [4]. Whereas, the original samples used by Wei et al. suffered of inhomogeneity effects, which have in general not been understood [5], the majority of samples used by many others [2,6] have mainly complicated the experiment due to inappropriate choices of the impurity characteristics [4].

In recent years, however, something remarkably universal has emerged in the actual shape of the magneto resistance data taken from the plateau–insulator (PI) transition of the lowest Landau level [6,7], that has not been observed in the original experiments of Wei et al. that were conducted on the plateau–plateau (PP) transitions of the higher Landau levels [1]. Whereas, the longitudinal resistance data associated with the PI transition generally follow an exponential law with varying magnetic field B [6,7], for a certain class of samples the Hall resistance has been found to be (almost) quantized throughout this transition [6]. In this communication we propose universal scaling functions for the conductance parameters that are based on new experimental data on the PI transition taken from a low-mobility InP/InGaAs heterostructure. These scaling functions encompass the universality statement made on the critical index κ [3]. However, unlike the numerical exponent values which are difficult to establish experimentally, we shall argue that the proposed scaling functions actually provide a more reliable and profound characterization of the quantum phase transition since they directly relate not only to the experiments conducted at finite T but also to the fundamental topological features of the underlying microscopic theory.

* Corresponding author. Tel.: +31 20 5255732; fax: +31 20 5255788.
E-mail address: devisser@science.uva.nl (A. de Visser).

0038-1098/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ssc.2006.01.016

2. Particle–hole symmetry

In magneto transport experiments one is generally faced with the long-standing problem of how to extract a local resistivity tensor ρ_{ij} from the measured macroscopic resistances R_{ij}. The experiment is usually conducted on samples prepared in the Hall bar geometry. Taking the x-direction as the direction of the electrical current then the resistivity components ρ_{xx} and ρ_{xy} are related to the measured resistances R_{xx} and R_{xy} according to:

$$R_{xx} = \frac{L}{W}\rho_{xx} \quad R_{xy} = \rho_{xy}. \tag{1}$$

Here, L/W is a geometrical factor with L denoting the 'length' and W the 'width' of the Hall bar, respectively. For a variety of reasons, however, the experiments generally do not provide unique and well-defined values of ρ_{xx} and ρ_{xy} [5]. In what follows we shall make use of the very special features of the PI transition that permit the observation of universal scaling functions that until now have been inaccessible. The solution lies—for the major part—in recognizing that the problem exhibits a fundamental symmetry [3,8,9] which we call particle–hole symmetry. To elucidate what it means we first recall the principles of scaling. Denoting the longitudinal and Hall resistivities of an ideal homogeneous sample by ρ_0 and ρ_H, respectively, then at sufficiently low T, these quantities with varying B and T become functions of a single scaling variable X only (we work in units of h/e^2)

$$\rho_0(B,T) = \rho_0(X), \quad \rho_H(B,T) = \rho_H(X), \tag{2}$$

where

$$X = \frac{(\nu-\nu_c)}{\nu_0(T)}; \quad \nu_0(T) = \left(\frac{T}{T_0}\right)^{\kappa}. \tag{3}$$

Here, $\nu = n_0/n_B$ equals the filling fraction of the Landau band with $\nu_c \approx 1/2$ denoting the critical value, n_0 is the electron density, $n_B = eB/hc$ the density of the fully occupied Landau level and T_0 stands for an arbitrary T scale. The conductivity components σ_0 and σ_H can by obtained as usual, by inverting the resistivity tensor:

$$\sigma_0 = \frac{\rho_0(X)}{\rho_0^2(X)+\rho_H^2(X)}, \quad \sigma_H = \frac{\rho_H(X)}{\rho_0^2(X)+\rho_H^2(X)}. \tag{4}$$

Particle–hole symmetry of the PI transition can now be expressed as follows:

$$\sigma_0(X) = \sigma_0(-X), \quad \sigma_H(X) = 1 - \sigma_H(-X). \tag{5}$$

As we shall see next, it is specifically this relation that enables us to disentwine the intrinsic transport properties of the PI transition from the sample dependent effects of macroscopic inhomogeneities.

3. The experiment

Our InP/InGaAs sample and set-up of the experiment conducted at high B are identical to those of van Schaijk et al. [7] in their studies of scaling of the PI transition. However, the new insights into the important role of sample inhomogeneities primarily emerge from the effects of a change in the polarity of the B field [10] that have previously not been investigated [7].

Fig. 1 shows the results for sweeps in both directions of the B field for different values of T. Upon reversing the direction of B at constant T we find that the ρ_{xx} data for the PI transition remain unchanged. The ρ_{xy} data, however, are strongly affected and the results for opposite B and low T ($T \leq 1.2$ K) display a remarkable symmetry about the plateau value $|\rho_{xy}| = h/e^2$. This result is clearly one of the strongest experimental features of the PI transition since it has previously been observed on different samples that do not provide access to scaling [6]. To understand the meaning of our results we split the ρ_{xy} data, at constant T, in different pieces

$$\rho_{xy}(B) = \rho_H(B) + \rho_{xy}^s(B), \tag{6}$$

where $\rho_H(B) = -\rho_H(-B)$ and $\rho_{xy}^s(B) = \rho_{xy}^s(-B)$. The relatively large component ρ_{xy}^s clearly indicates the effect of sample inhomogeneities, which may in principle be arbitrary complicated. In a simplest approach to the problem one can associate a geometrical significance with ρ_{xy}^s due to a misalignment of the sample contacts. To represent the effect we replace the geometry of Hall bar by that of a parallelogram obtained by rotating the y-axis of the $L\times W$ rectangular system over a small angle θ. Under these circumstances it is convenient to express the experimental resistivity tensor ρ_{ij} in terms of the intrinsic components ρ_0 and ρ_H as follows

$$\rho_{ij} = S_{ij}\rho_0(X) + \varepsilon_{ij}\rho_H(X). \tag{7}$$

Here, ε_{ij} is the usual antisymmetric tensor and the quantity S_{ij}, which we name the stretch tensor is given by

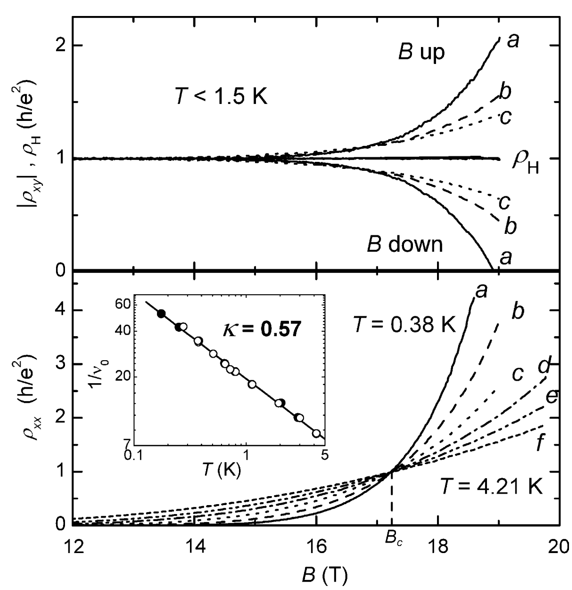

Fig. 1. Data for ρ_{xx} and $|\rho_{xy}|$ with varying B and for opposite field directions, taken from an InGaAs/InP heterojunction ($n = 2.2\times10^{11}$ cm^{-2}, $\mu = 16{,}000$ cm^2/V s). The letters $a,b,\ldots,f$ indicate $T = 0.38$, 0.65, 1.2, 2.1, 2.9 and 4.2 K, respectively. Inset: $1/\nu_0$ vs T for the PI transition. The closed and open symbols refer to opposite directions of the B field.

$$S_{ij} = \begin{bmatrix} \dfrac{\rho_c}{\cos\theta} & \tan\theta \\ -\tan\theta & \dfrac{\rho_c^{-1}}{\cos\theta} \end{bmatrix}. \quad (8)$$

We have introduced the symbol $\rho_c \approx 1$ to indicate the experimental uncertainties in the factor L/W. When written in this manner the problem of contact misalignment resembles in many ways that of an anisotropic magneto resistance tensor. Eqs. (7) and (8) are consistent with the observed symmetries under $B \to -B$:

$$\rho_{xx} = \frac{\rho_c}{\cos\theta}\rho_0(X); \quad \rho_{xy}^{s} = \tan\theta\rho_0(X). \quad (9)$$

Moreover, since there are no geometrical factors associated with the Hall resistance we make use of $\rho_H = 1$ at low T and immediately deduce the following important statement made by particle–hole symmetry (Eq. (5))

$$\rho_0(X) = \rho_0^{-1}(-X). \quad (10)$$

In the remainder of this communication we show that Eqs. (7)–(10) set the stage for a detailed analysis and understanding of the transport data. This analysis can be extended in several ways, including the effects of inhomogeneity in the electron density as well as important self-consistency checks. In this respect the PI transition is distinctly different from the PP transitions where the steps in ρ_H complicate the problem considerably [5].

4. The ρ_{xx} data

These data with varying B display a well-defined fixed point at $B_c = 17.2$ T ($\nu_c = 0.55$) where the various isotherms intersect (Fig. 1). This observation can be used to accurately determine the scaling variable X [7]. The data for $1/\nu_0(T)$ vs T are plotted on a log–log scale in the inset of Fig. 1 from which one extracts the critical exponent value $\kappa = 0.57 \pm 0.02$ as well as $T_0 = 200 \pm 25$ K, which are the same as those found earlier [7]. The ρ_{xx} data are, furthermore, consistent with Eqs. (7) and (8) and particle–hole symmetry (Eq. (10)) and the result for $\rho_0(X)$ for a large range in X can be expressed as follows

$$\rho_0(X) = \mathrm{e}^{-X - \mathcal{O}(X^3)}. \quad (11)$$

The term $\mathcal{O}(X^3)$ in the exponential is a small correction in the regime of actual interest $|X| \lesssim 1$ and a best estimate gives $0.002X^3$. The amplitude in Eq. (7) has been determined to be $\rho_c/\cos\theta = 1.0 \pm 0.1$ where the large error reflects the experimental difficulties in measuring the geometrical factor L/W of the Hall bar.

5. The ρ_{xy}^{s} data

The main features of the ρ_{xy}^{s} data are described by Eq. (9) with $\tan\theta \approx 0.1$ and $\rho_0(X)$ given by Eq. (11). The main difference, however, is the fixed point value ν_c which is slightly different for the ρ_{xy}^{s} and ρ_{xx} components. Since the measurements were conducted on different parts of the sample, the different values of ν_c can be attributed to small gradients in the electron density. To justify this claim we have assumed that the resistivity components are given by $\rho_0(X) = \mathrm{e}^{-X}$, $\rho_H = 1$ but with the filling fraction ν replaced by a spatially varying quantity $\nu(x,y) = \nu + \nu_x(x) + \nu_y(y)$ in the definition of X. This problem of spatially varying transport coefficients can be solved exactly. The final results can be written precisely in the form of Eq. (8) but with the stretch tensor now given by [5,10]

$$S_{ij} = \begin{bmatrix} \dfrac{\rho_c}{\cos\theta} & \tan\theta \ \exp\left(\dfrac{\varepsilon_x \nu}{\nu_0(T)}\right) \\ -\tan\theta \ \exp\left(\dfrac{\varepsilon_y \nu}{\nu_0(T)}\right) & \dfrac{\rho_c^{-1}}{\cos\theta} \end{bmatrix}. \quad (12)$$

Here, $\varepsilon_x = \pm \nu_x L/2\nu$ and $\varepsilon_y = \pm \nu_y W/2\nu$ are fixed quantities representing the relative uncertainties in the electron density in the x and y directions, respectively. The main point of this exercise is to demonstrate that the density gradients do not affect the critical behavior of the PI transition. For example, Eq. (12) implies that

$$\rho_{xy}^{s}(B,T) = \tan\theta \ \mathrm{e}^{\varepsilon_x \nu/\nu_0(T)} \quad \rho_0(X) = \tan\theta\rho_0(\tilde{X}), \quad (13)$$

where $\tilde{X}$ is the same as in Eq. (3) but with $\tilde{\nu}_c = \nu_c/(1+\varepsilon_x)$ and $\tilde{T}_0 = T_0(1+\varepsilon_x)^{1/\kappa}$ substituted for ν_c and T_0, respectively. From the difference $\nu_c - \tilde{\nu}_c$ we extract $\varepsilon_x \approx 0.02$. In Fig. 2(c) we plot the quantity $(1/6)[\varepsilon_x \nu_c/\nu_0(T)]^2$ vs T. The results are consistent with the general condition $(1/6)[\varepsilon_{x,y}\nu/\nu_0(T)]^2 \ll 1$ under which Eq. (12) is valid [5,10].

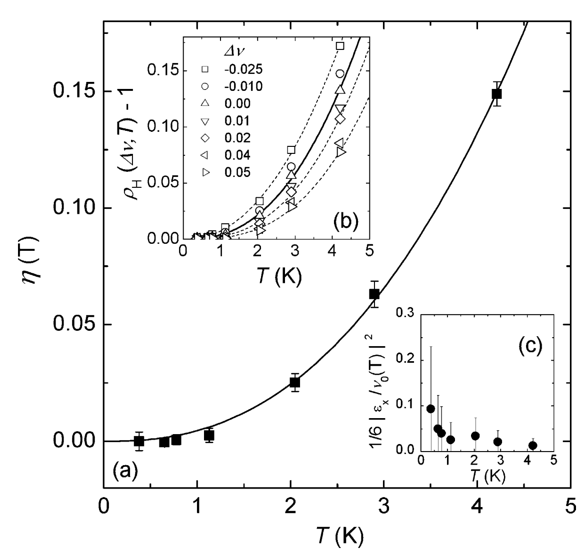

Fig. 2. (a) Collapse of the ρ_H data (see inset b) onto a single curve $\eta = (\rho_H - 1)/\rho_0(\tilde{X})$ vs T for different values of $\Delta\nu = \nu - \nu_c$. Here the $\rho_0(\tilde{X})$ has been taken from the ρ_{xy}^{s} data, Eq. (13). Solid line: $(T/T_1)^{y_\sigma}$ vs T with $y_\sigma = 2.43 \pm 0.08$ and $T_1 = 9.2 \pm 0.3$ K. (b) Data for $\rho_H - 1$ vs T for different values of $\Delta\nu$. (c) Data for $(1/6)[\varepsilon_x \nu_c/\nu_0(T)]^2$ vs T.

6. Corrections to scaling

To complete the analysis of the resistivity tensor, the small corrections to exact quantization $\rho_H = 1$ at higher T (not plotted in Fig. 1) are now addressed. The following expression (Fig. 2(a) and (b))

$$\rho_H = 1 + \eta(T)\rho_0(\tilde{X}), \quad \eta(T) = \left(\frac{T}{T_1}\right)^{y_\sigma} \tag{14}$$

with $y_\sigma = 2.43 \pm 0.08$ and $T_1 = 9.2 \pm 0.3$ K accurately describes the ρ_H data for $T < 4$ K. Here the quantity $\rho_0(\tilde{X})$ has the same meaning as in Eq. (13) and is obviously replaced by $\rho_0(X)$ in the final answer for ρ_H.

At this stage several remarks are in order. Notice first that the correction term $\eta(T)\rho_0(X)$ in Eq. (14) is precisely in accordance with the general expectations on the basis of critical phenomena theory. For example, under ordinary quantum Hall conditions (i.e. large values of X) Eq. (14) indicates that the corrections to exact quantization are exponential in T, which means that the low energy dynamics of the electron gas has a mass gap, as expected. At the quantum critical point $X = 0$, however, the corrections render algebraic in T indicating that the excitations are now massless. These various different statements become especially meaningful if one recognizes that the scaling corrections in Eq. (14) are actually obtained as a corollary of the renormalization theory of the quantum Hall effect [3,8,9]. To see this we express the conductivity components in terms of the independent scaling variables $\eta(T)$ and X. Working to linear order in $\eta(T)$ we obtain the following expressions

$$\sigma_0 = \frac{\rho_0}{\rho_0^2 + 1 + 2\eta\rho_0}, \quad \sigma_H = \frac{1 + \eta\rho_0}{\rho_0^2 + 1 + 2\eta\rho_0} \tag{15}$$

which generalize the statement of particle–hole symmetry (Eq. (5)). Moreover, when plotted as T-driven flow lines in the σ_0, σ_H conductivity plane (Fig. 3) we observe all the distinctly different and much sought after features of scaling that previously remained concealed in the experiments on the PP transitions [1].

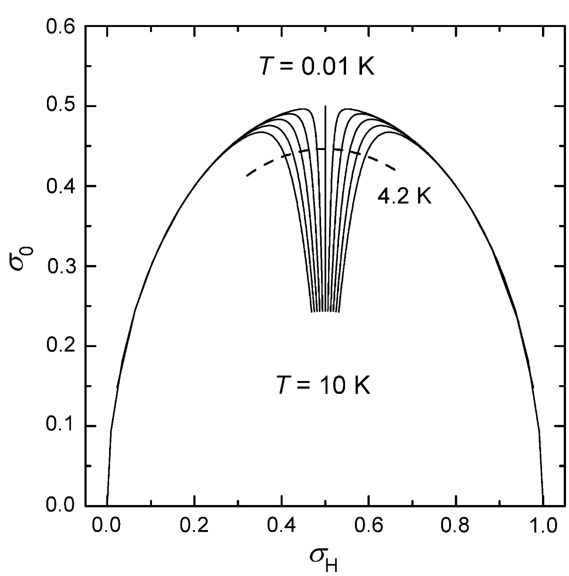

Fig. 3. Experimental T-driven flow lines with $0.01 < T < 10$ K for different values of ν near ν_c, according to Eq. (15).

7. Conclusions

For the first time universal scaling functions have been extracted from the experimental data. The most significant finding is Eq. (15), which has a remarkably general significance for a range of completely different physical systems. From the experimental point of view, the main differences are found in the expression for $\nu_0(T)$, which strongly depends on the type of disorder in the sample, given the range of experimental T. For example, in the case of smoothly varying potential fluctuations one finds the semiclassical result $\nu_0(T) = \alpha + \beta T$ [6] which gives no indication of the universal algebraic law of Eq. (3) which is expected at a much lower T only [4]. The exponent value $\kappa = 0.57$ taken from the PI transition is slightly different from the original results on the PP transition [1] which are complicated due to inhomogeneity effects [5]. From the theoretical side, Eq. (15) has interesting consequences for composite fermion theory [11] as well as the topological concept of an instanton vacuum [9], notably all the basic aspects of the quantum Hall effect are generically displayed by the latter [9,12]. Furthermore, our scaling results are very similar to those recently obtained from certain exactly solvable models of the θ term [12]. Eq. (15), therefore, clearly delineates the highly non-trivial and super universal consequences of topological concepts in quantum field theory that unify the robust quantization of the Hall conductance and the quantum critical behavior of the electron gas that is generally associated with the plateau transitions.

Upon completion of this work we learnt that detailed studies on κ are being performed by Li et al. [14]. These experiments seem to favor $\kappa \approx 0.42$ and possibly indicate that our slightly different value is a result of clustering effects in the alloy scattering. It is important to stress, however, that experimental κ values cannot be used for the purpose of justifying Fermi liquid ideas [13] since the problem with long ranged (Coulomb) interactions is known to be in a different universality class, involving fundamentally different symmetries ($\mathscr{F}$ invariance) as well as a different dimensionality [8].

Acknowledgements

The research was supported in part by FOM and INTAS. One of us (AP) is indebted to D.C. Tsui for sharing his results prior to publication.

References

[1] (a) H.P. Wei, D.C. Tsui, M.A. Paalanen, A.M.M. Pruisken, Phys. Rev. Lett. 61 (1988) 1294;
(b) H.P. Wei, D.C. Tsui, A.M.M. Pruisken, Phys. Rev. B 33 (1985) 1488.

[2] (a) S. Koch, R.J. Haug, K. von Klitzing, K. Ploog, Phys. Rev. B 43 (1991) 6828;
(b) S. Koch, R.J. Haug, K. von Klitzing, K. Ploog, Phys. Rev. B 46 (1992) 1596.
[3] A.M.M. Pruisken, Phys. Rev. Lett. 61 (1988) 1297.
[4] A.M.M. Pruisken, B. Škorić, M. Baranov, Phys. Rev. B 60 (1999) 16838.
[5] (a) B. Karmakar, M.R. Gokhale, A.P. Shah, B.M. Arora, D.T.N. de Lang, A. de Visser, L.A. Ponomarenko, A.M.M. Pruisken, Physica E 24 (2004) 187;
(b) L.A. Ponomarenko, D.T.N. de Lang, A. de Visser, V.A. Kulbachinskii, G.B. Galiev, H. Künzel, A.M.M. Pruisken, Solid State Commun. 130 (2004) 705.
[6] (a) M. Hilke, D. Shahar, S.H. Song, D.C. Tsui, Y.H. Xie, D. Monroe, Nature 395 (1988) 675;
(b) D. Shahar, D.C. Tsui, M. Shayegan, J.E. Cunningham, E. Shimshoni, S.L. Sondhi, Solid State Commun. 102 (1997) 817;
(c) D. Shahar, M. Hilke, C.C. Li, D.C. Tsui, S.L. Sondhi, J.E. Cunningham, M. Razeghi, Solid State Commun. 107 (1988) 19.
[7] R.T.F. van Schaijk, A. de Visser, S.M. Olsthoorn, H.P. Wei, A.M.M. Pruisken, Phys. Rev. Lett. 84 (2000) 1567.
[8] (a) A.M.M. Pruisken, M.A. Baranov, I.S. Burmistrov, cond-mat/0104387.
(b) M.A. Baranov, I.S. Burmistrov, A.M.M. Pruisken, Phys. Rev. B 66 (2002) 075317.
[9] For a recent review see A.M.M. Pruisken, I.S. Burmistrov, Ann. Phys. 316 (2005) 285 ((cond-mat/0407776) and references therein).
[10] (a) D.T.N. de Lang, L. Ponomarenko, A. de Visser, C. Possanzini, S.M. Olsthoorn, A.M.M. Pruisken, Physica E 12 (2002) 666;
(b) D.T.N. de Lang, PhD Thesis, University of Amsterdam, 2005.
[11] (a) S. Kivelson, D.H. Lee, S.C. Zhang, Phys. Rev. B 46 (1992) 2223;
(b) A.M.M. Pruisken, M. Baranov, B. Škorić, Phys. Rev. B 60 (1999) 16807.
[12] A.M.M. Pruisken, M.A. Baranov, M. Voropaev, JETP Lett. 82 (2005) 150 (cond-mat/0101003).
[13] See e.g. (a) D.G. Polyakov, B.I. Shklovskii, Phys. Rev. Lett. 73 (1994) 1150; (b) S.R.E. Yang, A.H. MacDonald, B. Huckestein, Phys. Rev. Lett. 74 (1995) 3229;
(c) B. Huckestein, M. Backhaus, Phys. Rev. Lett. 82 (1999) 5100;
(d) F. Hohls, U. Zeitler, R.J. Haug, Phys. Rev. Lett. 86 (2001) 5124.
[14] W. Li, G.A. Csáthy, D.C. Tsui, L.N. Pfeiffer, K.W. West, Phys. Rev. Lett. 94 (2005) 206807.