Nonmonotonic temperature dependence of the resistivity of $p$-Ge/Ge$_{1-x}$Si$_x$ in the region of the metal–insulator transition

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INTRODUCTION

In disordered 2D systems at low temperatures there are two types of quantum corrections $\delta \sigma_0 = \delta \sigma_{nl} + \delta \sigma_{ee}$ to the Drude conductivity $\sigma_0 = e^2/h(k_Fl)$; $\delta \sigma_{nl}$ is the correction due to inertial effects in the scattering of the electron waves on impurities (weak localization), and $\delta \sigma_{ee}$ is the correction due to the disorder-modified electron–electron interaction.\(^1\)\(^2\)

In weakly disordered systems with $k_Fl > 1$ these corrections are small in the parameter $(k_Fl)^{-1}$ ($l$ is the mean free path) and depend logarithmically on temperature.

Experiments to detect the so-called metal–insulator transition from the change in the carrier density in 2D semiconductor structures with high mobility have stimulated a substantial advance in the theory of electron–electron interaction effects.\(^6\)\(^7\)

For example, the linear growth of the resistivity $\rho$ with temperature in Si-MOSFET structures with high carrier mobility at large values $\sigma_0 \gg e^2/h$, which for the past decade has been considered to be a manifestation of an “anomalous metallic” state, is now interpreted as being due to an electron–electron interaction effect in the ballistic regime.\(^8\)

However, the nonmonotonic temperature dependence of $\rho(T)$ observed near the proposed metal–insulator transition ($\sigma_0 \approx e^2/h$) still does not have a generally accepted explanation. This has stimulated our investigations into multilayer $p$-Ge/Ge$_{1-x}$Si$_x$ heterostructures.

Suppression of the low-temperature conducting phase by a magnetic field parallel to the 2D layer (positive magnetoresistance) has been observed repeatedly for high-mobility Si-MOSFETs\(^9\)-\(^15\) and $p$-GaAs heterostructures\(^16\),\(^17\) such behavior is explained either by the “complete polarization” of the electron (hole) gas\(^12\)-\(^14\),\(^17\),\(^18\) or (at low fields) by the Zeeman effect in the quantum correction owing to the electron–electron interaction in both the diffusion\(^19\) and ballistic\(^8\),\(^15\) regimes.

We have carried out studies in a magnetic field perpendicular to the 2D layer, where together with the Zeeman level splitting it is necessary to take weak localization effects into account. The hole gas in the Ge quantum wells for the $p$-Ge/Ge$_{1-x}$Si$_x$ heterostructures studied is described by the Luttinger Hamiltonian with a highly anisotropic $g$ factor in respect to the mutual orientation of the magnetic field and the 2D plane. At the bottom of the lower spatial subband $g_\perp = \pm 6\kappa$ (where for Ge the Luttinger parameter $\kappa$ is 3.4)\(^20\) for the perpendicular magnetic field and $g_\parallel = 0$ for the parallel.\(^21\),\(^22\) For interpretation of our experimental $\rho(B,T)$ curves in the samples near the proposed metal–insulator phase transition we invoked a model used for semiconductor...
ing 2D systems with high mobility.\textsuperscript{10,15,19,23,24}

**EXPERIMENTAL RESULTS AND DISCUSSION**

Measurements of the galvanomagnetic effects in multilayer heterostructures of \( p \)-type Ge/Ge\(_{1-x}\)Si\(_x\) were made in magnetic fields up to 5 T at \( T=0.3\)–4.2 K. For a sample \( T! \) with a carrier density of \( 1.2\times10^{11} \) cm\(^{-2} \) and mobility \( \mu_p=4\times10^{3} \) cm\(^2\)/(V\(\cdot\)s) (parameter \( e_F\tau\hbar=0.75 \)), nonmonotonic low-temperature behavior of the resistivity is observed (Fig. 1a): \( \rho(T) \) increases with decreasing temperature from 4.2 to 1.5 K (localization) and then \( \rho(T) \) decreases as \( T \) is lowered further from 1.5 to 0.3 K (antilocalization).

In the antilocalization region for \( T!1 \) K the conductivity depends logarithmically on temperature (Fig. 1b). In the whole temperature interval positive magnetoresistance is observed, increasing sharply with decreasing \( T \) (Fig. 2a). At low temperatures \( T!1 \) K in fields \( B\leq0.3 \) T the magnetoresistivity \( \Delta\rho_{xx} \) is an almost universal function of the ratio \( B/T \) (Fig. 2b).

The observed \( \rho(B,T) \) curves can be compared with the quantum corrections to the two-dimensional conductivity due to the weak localization effects (\( \delta\sigma_{ee} \)) and to electron–electron interaction (\( \delta\sigma_{ee} \)). For the electron–electron interaction effects in the diffusion regime \( k_B\tau/\hbar\ll1 \) we have\textsuperscript{1,2}

\[
\delta\sigma_{ee}(B,T) = \delta\sigma_{ee}(0,T) + \delta\sigma_{s}(b),
\]

where

\[
\delta\sigma_{ee}(0,T) = \frac{e^2}{2\pi^2\hbar}(1 - 3\lambda)\ln\frac{k_B\tau}{\hbar},
\]

\[
\delta\sigma_{s}(b) = -\frac{e^2}{2\pi^2\hbar}G(b) \left( b = \frac{g\mu_Bb}{k_B\tau} \right).
\]

The first term in front of the logarithm in Eq. (2) corresponds to the exchange part of the electron–electron interaction, while the second term corresponds to the Hartree contribution (triplet channel). Here

\[
\lambda = \frac{1 + \gamma_2}{\gamma_2} - \ln(1 + \gamma_2) - 1,
\]

where \( \gamma_2 \) is the Fermi-liquid interaction parameter.\textsuperscript{25}

The electron–electron contribution of the magnetic field is given as a function of the ratio \( B/T \) by expression (3), where \( G(b) \) is a known function describing the positive magnetoresistance due to the splitting of the electron energy levels,\textsuperscript{1,26,27} and \( g=20.4 \) for a 2D hole gas in Ge for \( e_F\rightarrow0 \).

For weak localization effects\textsuperscript{28}
The prevalence of the first effect. For example, let us give the expression for 
\[ \delta \sigma = \delta \sigma_{ee} + \delta \sigma_{wl} \] 
low fields \( B \ll B_z \) \( = k_B T / g \mu_B \), \( B \ll B_\varphi \). 
\[ \delta \sigma(B, T) = \frac{e^2}{2 \pi^2 \hbar} \left[ -0.091 \gamma_2 (1 + \gamma_2) + 0.042 \left( \frac{B_z}{B_\varphi} \right)^2 \right] \times \left( \frac{B}{B_z} \right)^2, \] 
where \((f \varphi = 2)\) behavior with decreasing temperature.25 (Fig. 3). As was shown in Ref. 24, such a renormalization is especially important in the region of the metal–insulator transition, which is determined by the condition \( e_F / \hbar \sim 1 \). We assume that the nonmonotonic \( \rho(T) \) dependence observed by us is due to just such a renormalization of the parameter \( \gamma_2 \) and, as a result, to a change in sign of the coefficient \((p + 1 - 3 \lambda)\) at \( T \approx 1.5 \) K, although we have been unable to describe the effect quantitatively.

**CONCLUSIONS**

Thus the observed nonmonotonic behavior of \( \rho(T) \) and, specifically, the transition from insulating \((d \rho/dT > 0)\) to "metallic" \((d \rho/dT < 0)\) behavior with decreasing temperature is attributed by us to enhancement of the role of the triplet channel in the quantum correction to the conductivity due to the electron–electron interaction. The increase of the contribution of the triplet channel with decreasing temperature is apparently due to the renormalization of the electron–electron coupling parameter predicted in the Finkelstein

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**FIG. 4.** Temperature dependence of the magnetoresistivity in fixed magnetic fields.

![Temperature dependence of the magnetoresistivity in fixed magnetic fields](image-url)
theory,\textsuperscript{25} which is especially substantial for 2D systems in the vicinity of the concentration-induced metal–insulator transition \((\varepsilon_F \tau \hbar \approx 1)\). The Zeeman splitting of the electron energy levels in a magnetic field leads to effective suppression of the triplet channel, thus restoring the insulating behavior of \(\rho(T)\) down to the lowest temperatures (Fig. 4).

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\textsuperscript{1} The theoretical parameters of the sample were: number of periods \((\text{Ge + GeSi}) N = 15\); quantum well (Ge layer) width \(d_w = 80 \text{ Å}\), and barrier (GeSi layer) width \(d_b = 120 \text{ Å}\).

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