STATIC SCALING IN HEAVY FERMIONS

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The usual assumptions of critical scaling near a transition at \( T_c \) lead, when \( T_c \) is negative, to solutions which are acceptable thermodynamically and seem appropriate to describe physical situations such as heavy fermions. Illustrations are given with the specific heats of \( \text{UBe}_13 \) and \( \text{UPt}_3 \).

The standard theory of continuous phase transitions expresses the Gibbs potential of \( N \) interacting particles of moment \( \mu \) as that of \( N/n_0 \) independent particles of size \( \xi \) in dimension \( d \) and of moment \( \mu_{\text{eff}} = \mu (\xi/\xi_0)^{d-u}(u \leq d) \) so that \([1,2]\)

\[
G/T \sim \xi^{d-g(\xi^{d-u}\eta)} \quad \text{where} \quad \eta = H/T.
\]

(1)

The static scaling hypothesis states that the coherence length \( \xi \) is an analytical function of the reduced interaction \( K = J/T \) which diverges like

\[
(\xi/\xi_0)^d = \left( (K_c - K)/K_c \right)^{-\theta/T_c} = (1 - T_c/T)^{-\theta/T_c}
\]

(2)

when \( K \) reaches by lower values a convergence radius \( K_c \) which defines \( T_c \). We have introduced the parameter \( \theta \)

\[
\theta/T_c = d\nu = 2 - \alpha = \gamma + 2\beta
\]

(3)

instead of the usual exponent \( \nu \). Differentiating \( G \) with respect to \( T \) we have for \( \eta = 0 \)

\[
\mathcal{S} = \delta G/\delta T \sim (1 + (\theta - T_c)/T_c)^{-1} \left( 1 - T_c/T \right)^{\theta/T_c-1}.
\]

(4)

\[
C_p T^2/(R \ln(2S + 1)) = \theta(\theta + T_k)(1 + T_k/T)^{-2-\theta/T_k} = f(\theta, T_k)
\]

(7)

which becomes a Schottky anomaly \( C_p T^2/(R \ln(2S + 1)) = \theta^2 \exp \left(-\theta/T\right) \) when \( T_k \) tends to zero.

We have evaluated the amplitude of the coefficient \( A \) which factorizes the specific heat in eq. (5) by imposing that the total entropy \( \mathcal{S}(T = \infty) - \mathcal{S}(T = 0) = R \ln(2S + 1) \). This is possible if \( T_c < 0 \) as the formula remains analytic down to \( T = 0 \). This leaves only two parameters \( \theta \) and \( T_k \) which can be determined from the knowledge of two points or from the knowledge of e.g. \( C_p(T_{\text{max}}) \) at \( T_{\text{max}} \) where the specific heat is maximum. We find that \( T_{\text{max}} = \theta/2 \) and

\[
C_p(T_{\text{max}})/(R \ln(2S + 1)) = 4(1 + T_k/\theta)(1 + 2T_k/\theta)^{-2-\theta/T_k}
\]

(8)

which is represented in fig. 1.

From the knowledge of \( C_p(T_{\text{max}}) = 2 \) J/mol K at \( T_{\text{max}} = 10 \) K in \( \text{CeRu}_2\text{Si}_2 \) we deduced \( \theta = 20 \) K.

Fig. 1. The amplitude \( C_p(T_{\text{max}}) \) of the specific heat at \( T_{\text{max}} = \theta/2 \) when it is maximum is shown vs. \( \theta/T_k \) for a total entropy \( \mathcal{S} = R \ln 2 \). The arrows point to \( \theta/T_k \) values appropriate for \( \text{UBe}_13 \) and \( \text{UPt}_3 \).
and $T_k = 13.3$ K from the plot of fig. 1. With these values we could account for the specific heat data within better than 5%. Similar results were obtained with CeCu$_6$ and CePd$_3$B$_2$ [1,2] supporting the conjecture that the case $T_c < 0$ is well adapted to describe heavy fermions. In general we have found that eq. (6) could give a much better account of the data than the available theories. Admittedly though, as any scaling theory the model does not contain a microscopical information. Also it does not solve a number of difficulties which are also difficulties for the available theories. For example the fig. 2 shows our data in UBe$_{13}$ between 0.4 and 1.2 K in a field of 8 T which was applied to suppress superconductivity [3]. The normal specific heat appeared to be field independent up to these values. The data were adjusted around 1 K with those of ref. [4] which involved that our results should be multiplied by a factor 1.11. The specific heat between 0.2 and 17 K is compared in the fig. 3 with the predictions of eq. (8) which we used twice to fit both the low temperature data and the residual Schottky-like tail. We found $C_p T^2/(R \ln 2) \sim 0.75f(\theta_1,T_{k1}) + f(\theta_2,T_{k2})$ where $f(\theta T_k)$ is defined in eq. (7) and where $\theta_1 = 2T_{max} = 5.8$ K with $T_{k1} = 4.1$ K and $\theta_2 = 46$ K with $T_{k2} = 4.5$ K. The problem is the factor 3/4 which is necessary to adjust the actual $C_p(T_{max})$ value to that which is expected from fig. 1 for the same best value of the $T_k/\theta \sim 4/3$ ratio. This means that we find only an entropy of 0.75$R \ln 2$ below the theoretical curve. We have not the same problem with the high temperature residual tail which announces a first Schottky-like anomaly with entropy $R \ln 2$ and $T_{max} \sim 23$ K.

Fig. 3 shows our data in UPt$_3$ between 2 and 20 K [5] and the theory for $\theta = 2T_{max} = 46$ K [6] with $\theta/T_k = 1.08$. The fit looks satisfactory but it corresponds to a total entropy $2R \ln 2$ and does not account for the detail of the evolution around 16 K which suggests that we may have two slightly shifted contributions.

References