

Muon Spin Rotation/Relaxation/Resonance



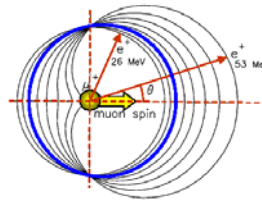
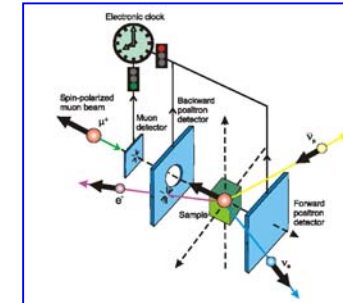
μ SR

- magnetic systems
- superconductors
- semiconductors
- chemistry

picture thanks to J.E. Sonnier
<http://musr.org/intro/musr/musrbrochure.pdf>

Contents

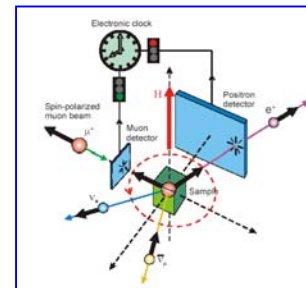
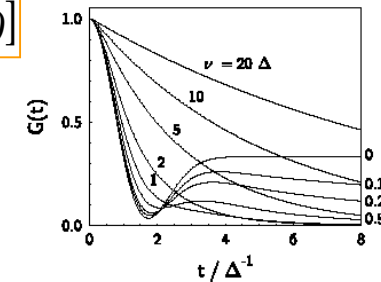
- Principles and techniques



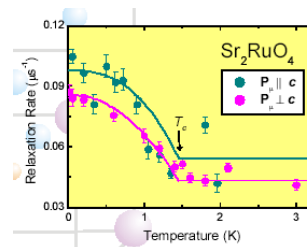
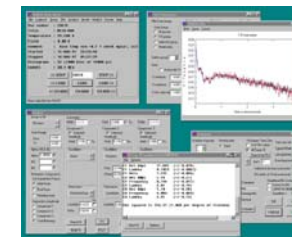
- Muon spin relaxation

$$N(t) = B + N_0 e^{-t/\tau_\mu} [1 + A(t)]$$

- Muon spin rotation



- Examples and analysis

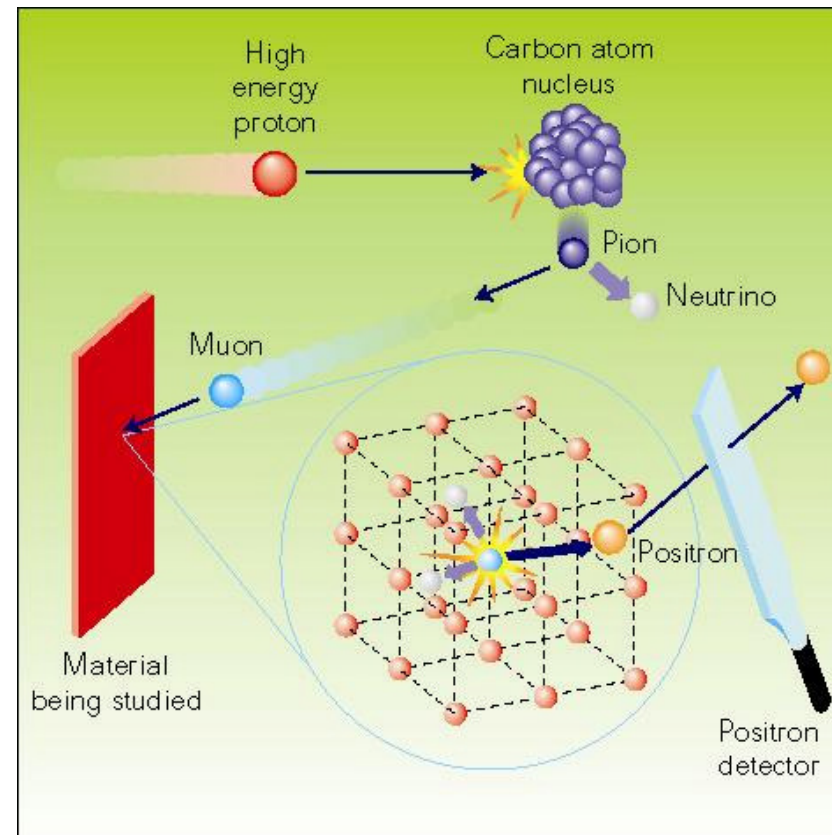


Resources and thanks to.....

- Websites
ISIS (<http://www.isis.rl.ac.uk/muons>)
PSI (<http://lmu.web.psi.ch>)
TRIUMF (<http://musr.org/>)
- Part of the pictures and course material is taken from the ISIS site, see:
[Lecture Notes “Training Course in Pulsed Muon Techniques”](#)
[and “the \$\mu\$ SR Brochure”](#)
- Special thanks to J.E. Sonnier, S.J. Blundell, B. Cywinsky and many others



1. Principles and techniques



What is a muon?

- elementary particle
 μ^+ is “light proton”
- muons are produced by pion decay

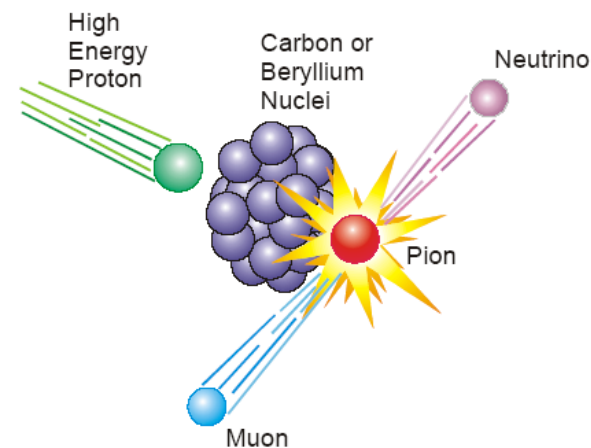
$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

- beam of spin polarized muons μ^+

	charge	spin	mass	moment	$\gamma / 2\pi$ (MHz T ⁻¹)	lifetime (μ s)
e	$\pm e$	1/2	m_e = 0.51 MeV	$657 \mu_p$	28×10^3	∞
μ	$\pm e$	1/2	$207 m_e$ = 105.7 MeV	$3.18 \mu_p$	135.5	2.19
p	$\pm e$	1/2	$1836 m_e$ = 938 MeV	μ_p	42.6	∞

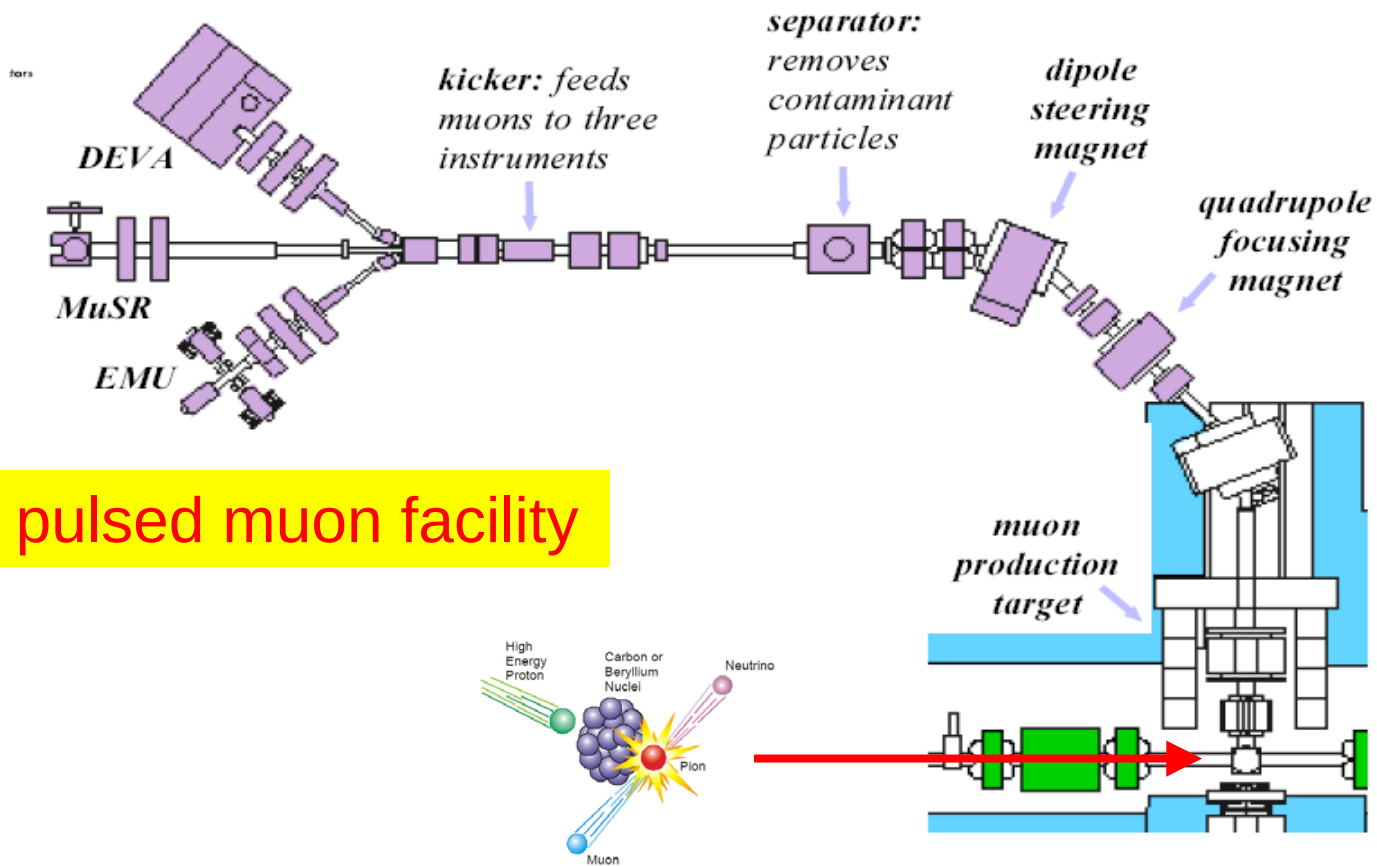
proton hits target



μ SR facilities



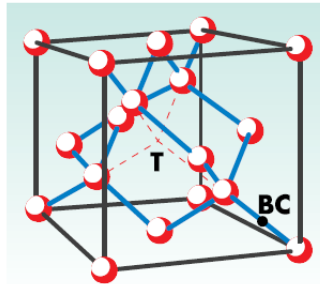
Production of muon beams



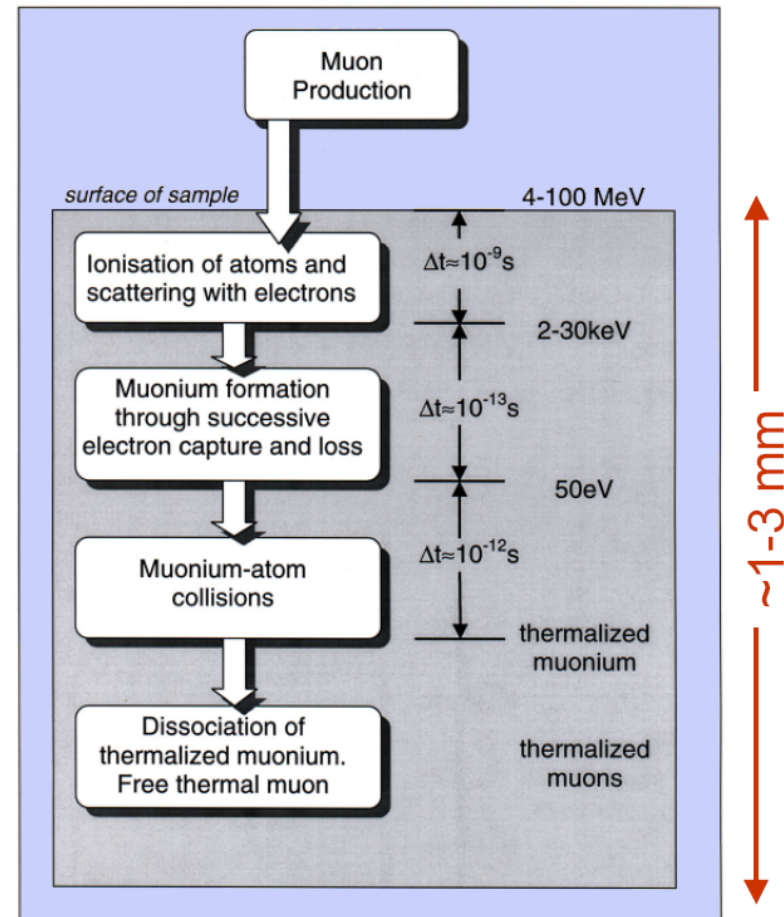
ISIS pulsed muon facility

Muon implantation

- muons (μ^+) are implanted in the sample and stop at interstitial site

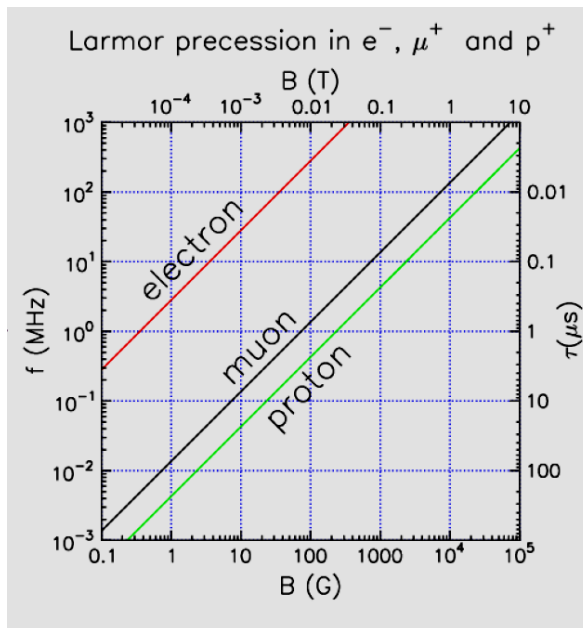


- rapid and without loss of polarization



Muon precession

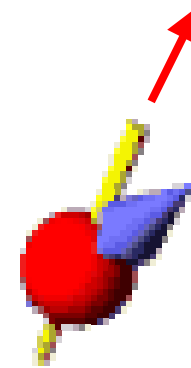
- beam of spin polarized muons μ^+ with polarization $\mathbf{P}_\mu(0)$
- μ^+ precesses around the local field with Larmor frequency ω_μ



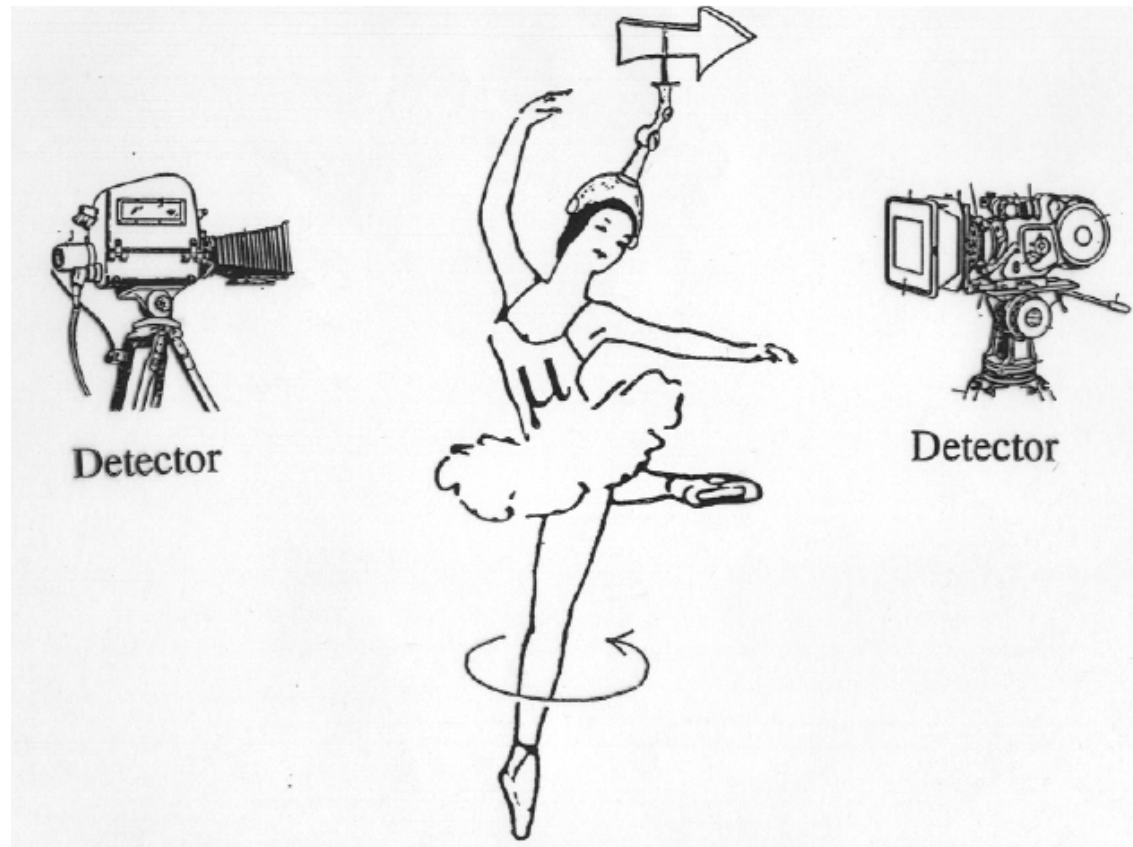
$$\omega_\mu = \gamma_\mu B_{loc}$$

gyromagnetic ratio
 $\gamma_\mu = 135.5 \text{ MHz/T}$

local
magnetic
field

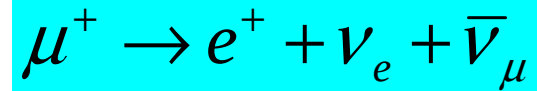


How to detect the muon spin?



Muon decay

- muon decays
life time $\tau_\mu = 2.19714 \mu\text{s}$

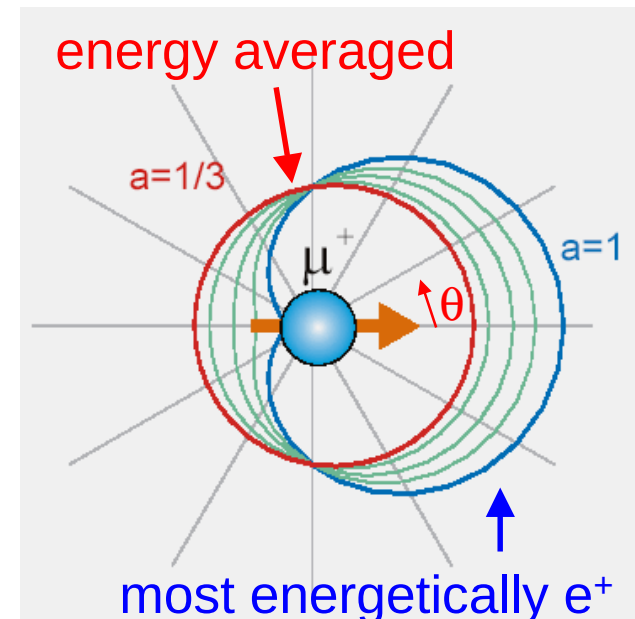


- positrons are emitted preferentially in direction muon spin

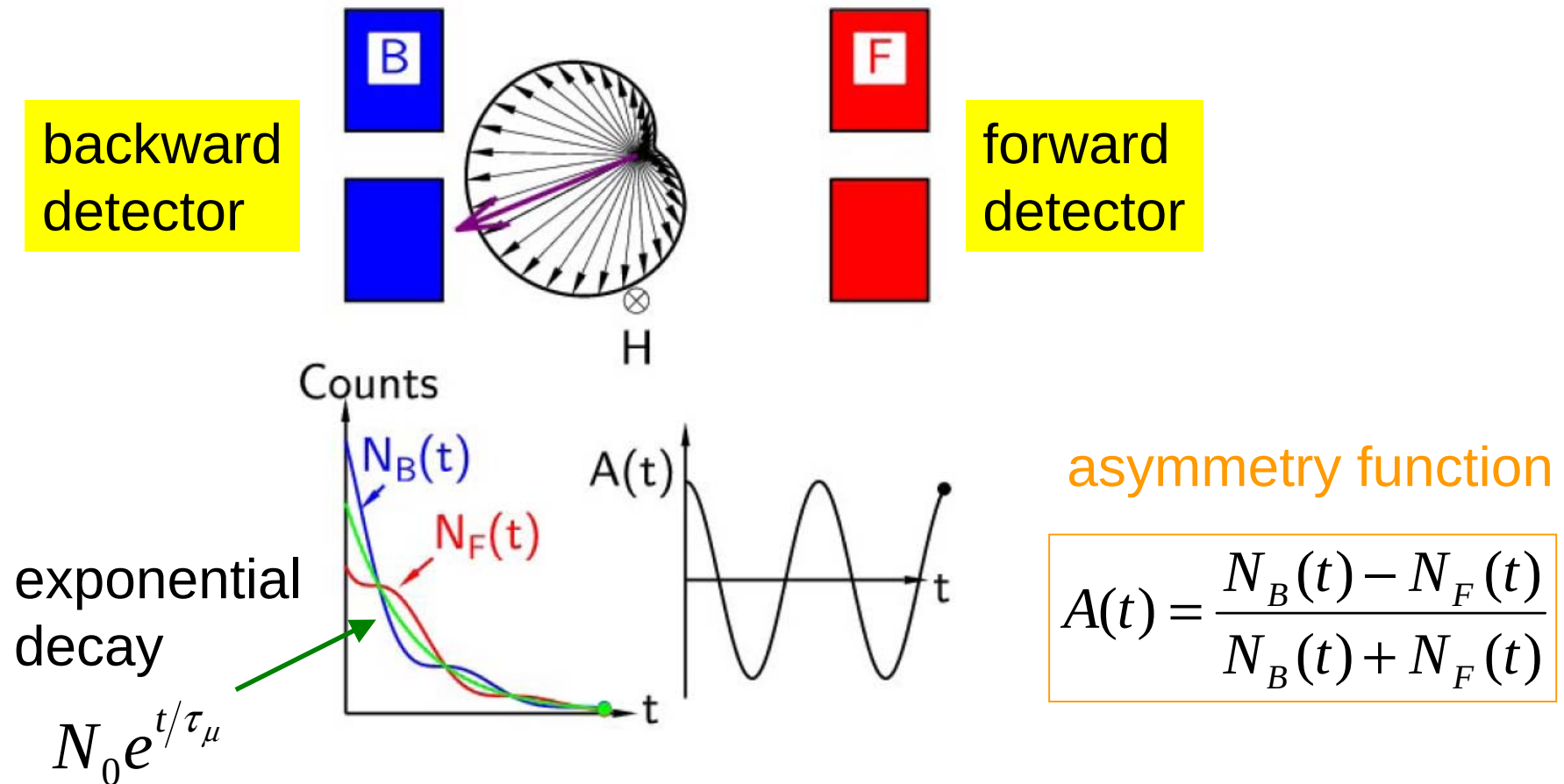
- decay asymmetry

$$W(\theta) = 1 + A \cos \theta$$

$$A = a P_\mu(0); a \sim 1/3$$



Detection of positrons



μ^+ depolarization function

- positron distribution, time histogram

$$N(t) = B + N_0 e^{-t/\tau_\mu} [1 + A(t)]$$

$$A_{F,B} \sim 0.25$$

$$N_{F,B}^{e^+}(t) = B_{F,B} + N_{F,B}^0 e^{-t/\tau_\mu} [1 \mp A_{F,B} G(t)]$$

$$G(t) = A(t) / A$$

- $G(t)$: normalized μ^+ spin autocorrelation function or μ^+ depolarization function

$$G(t) = \frac{\langle \mathbf{S}(t) \cdot \mathbf{S}(0) \rangle}{S(0)^2}$$

$$G(t) = \frac{P_\mu(t)}{P_\mu(0)} = \frac{\mathbf{P}_\mu(t) \cdot \mathbf{P}_\mu(0)}{P_\mu(0)^2}$$

$$P_\mu(0) = |\mathbf{P}_\mu(0)| \sim 1 = \text{beam polarization}$$

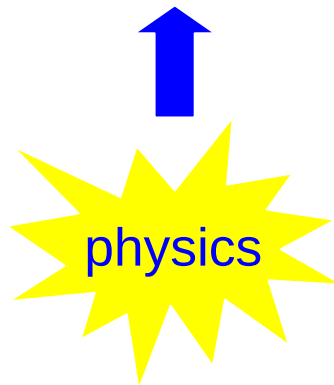
$$\mathbf{P}_\mu(t) = \text{time evolution polarization implanted muons}$$

Example ferromagnet

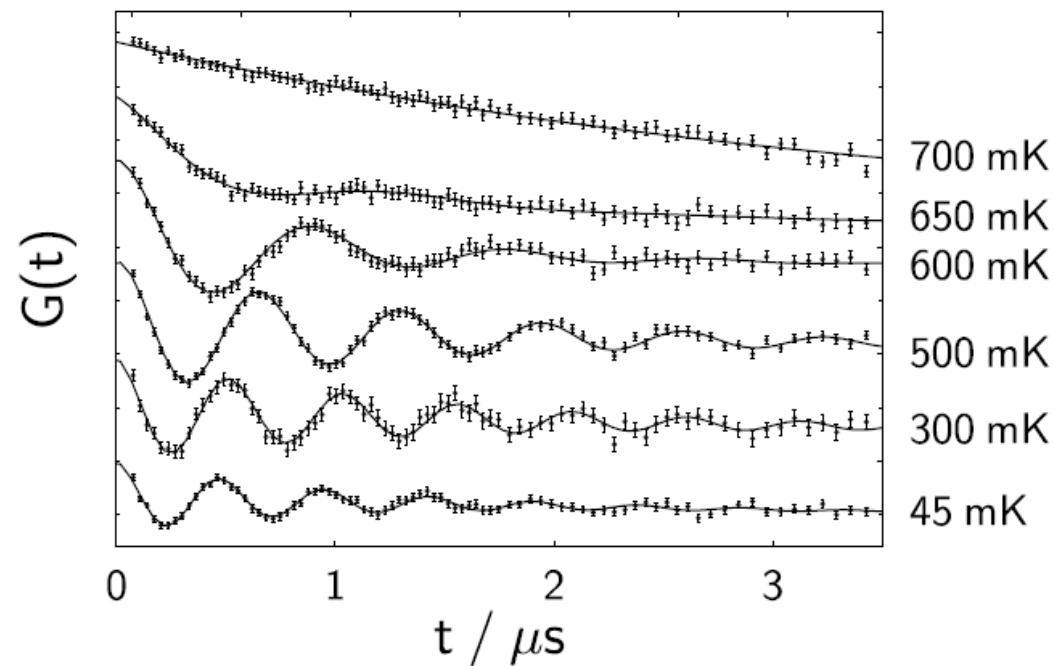
- μ^+ depolarization function $G(t)$

$$G(t) \sim e^{-\frac{1}{2}\sigma^2 t^2} \cos(\omega t)$$

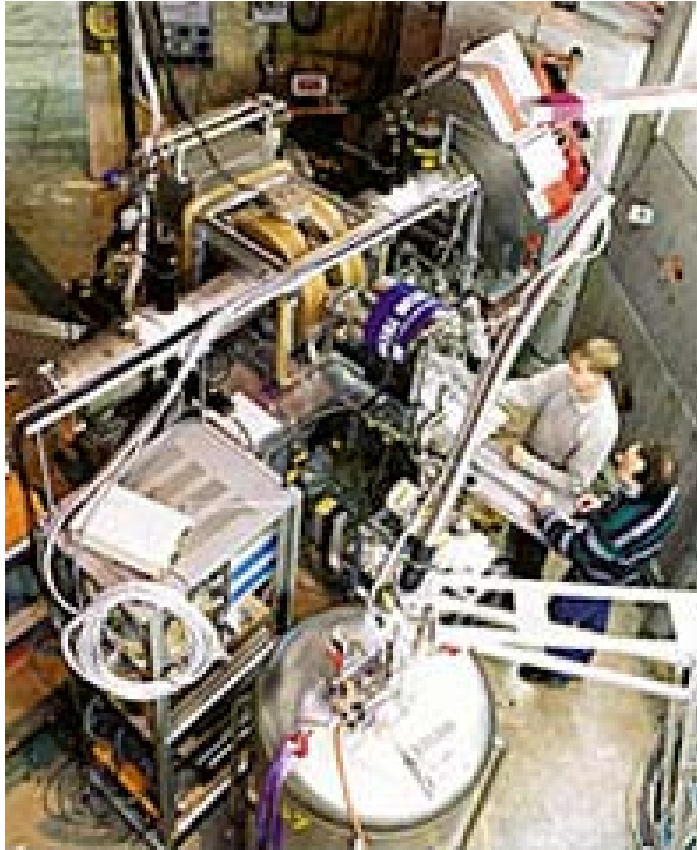
Curie temperature
 $T_C = 0.7$ K



zero-field muon
precession in an
organic ferromagnet
(Blundell et al., 1995)



Real spectrometers



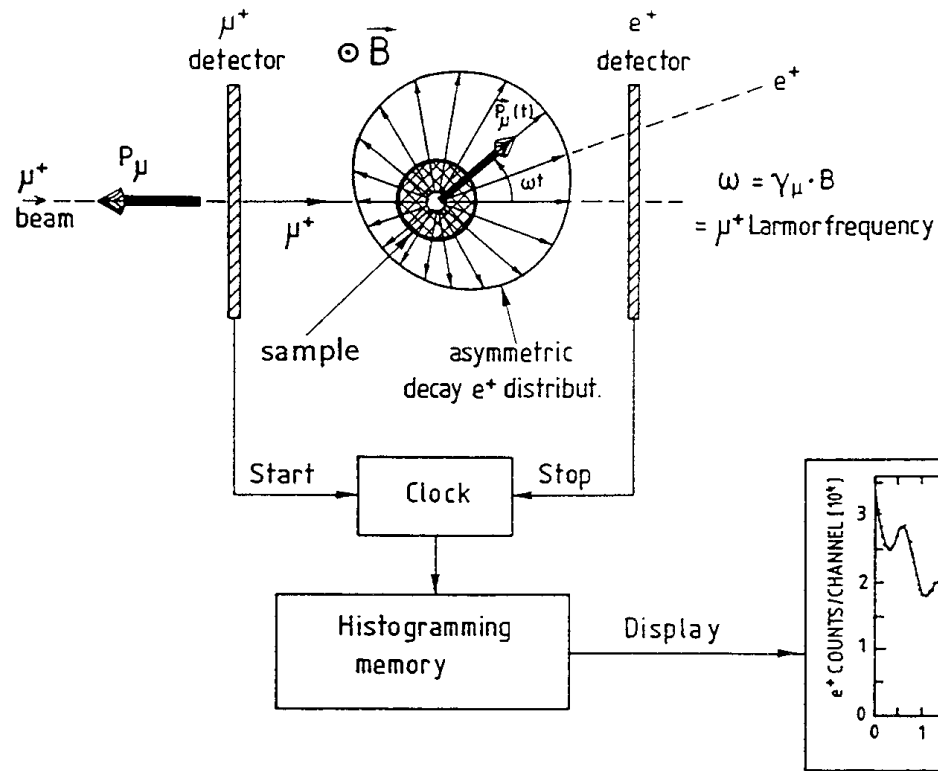
GPS at PSI



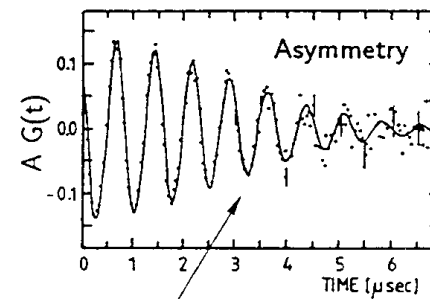
LAMPF at Triumf

Re-cap

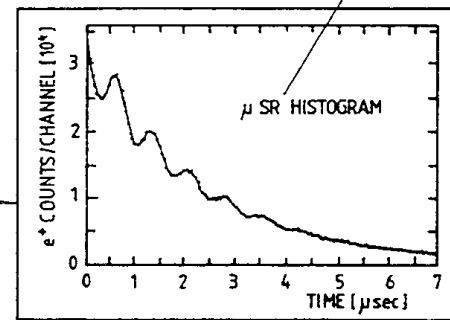
transverse field setup



asymmetry



Analysis

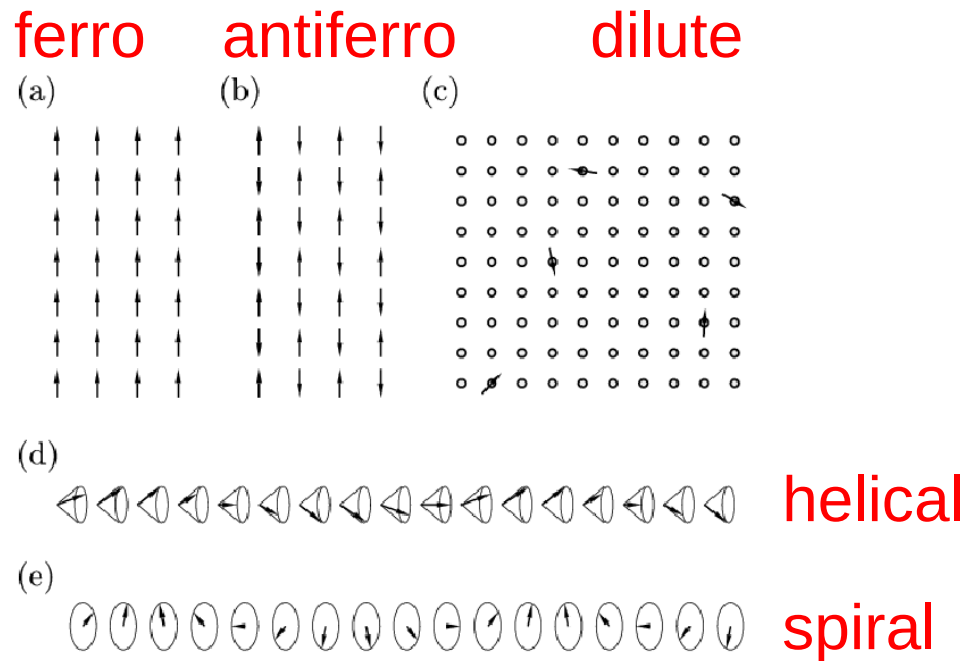


physics: $B_{\text{loc}}, \Delta B$

time histogram

$$N(t) = B + N_0 e^{-t/\tau_\mu} [1 + AG(t)]$$

Muon as magnetic probe in different materials



- nuclear moments
- electronic moments

- static moments
- fluctuating moments

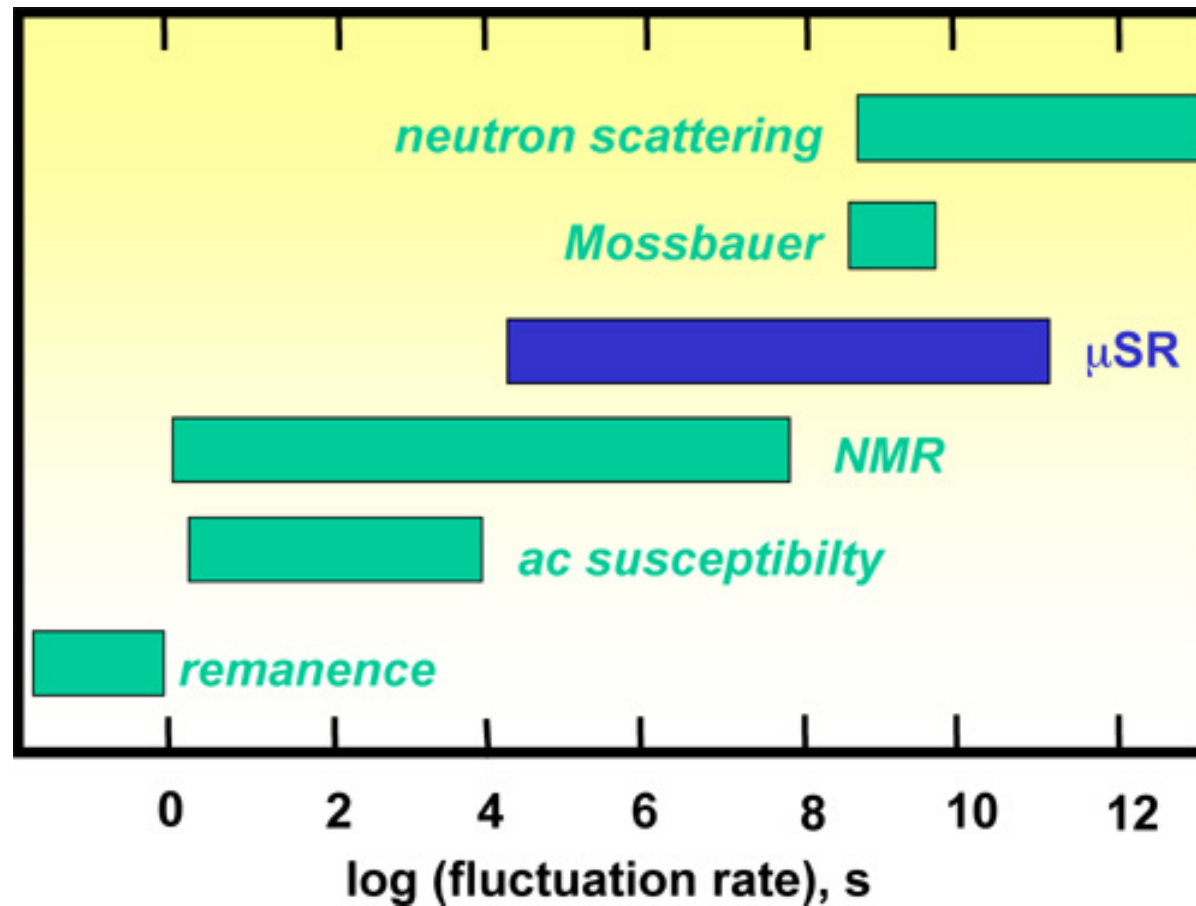


- various spin structures
- spin glasses (randomness)
- variety of field distributions



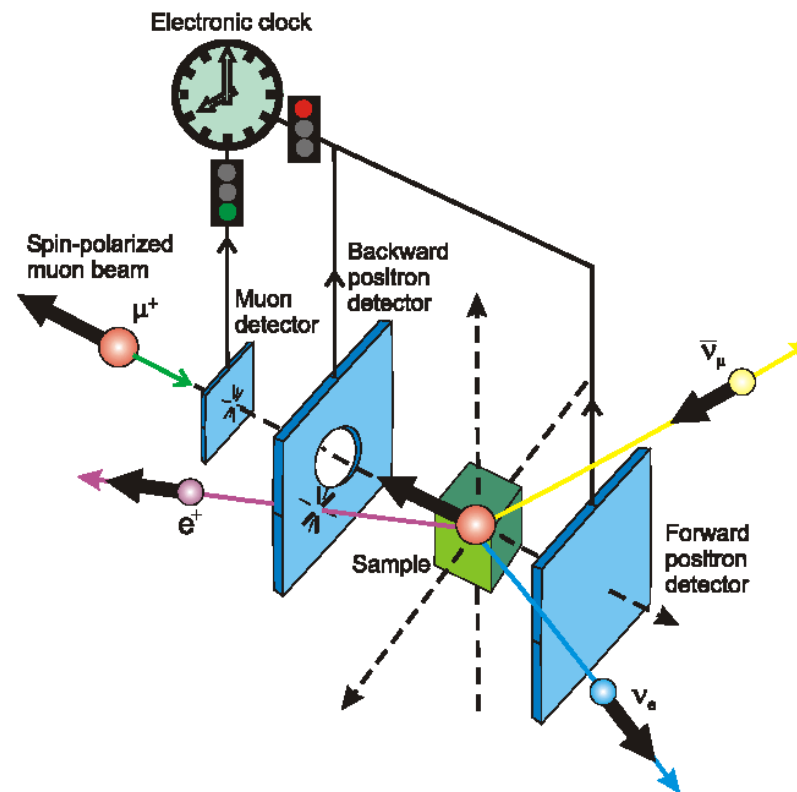
dedicated $G(t)$
for each problem

μ SR compared to other techniques



2. Muon spin relaxation

- Zero or longitudinal field



Gaussian and Lorentzian distributions

- static Gaussian field distribution, width Δ
example: nuclear dipole fields

$$f^G(B_i) = \sqrt{\frac{1}{2\pi}} \frac{\gamma_\mu}{\Delta} \exp\left(-\frac{\gamma_\mu^2 B_i^2}{2\Delta^2}\right)$$

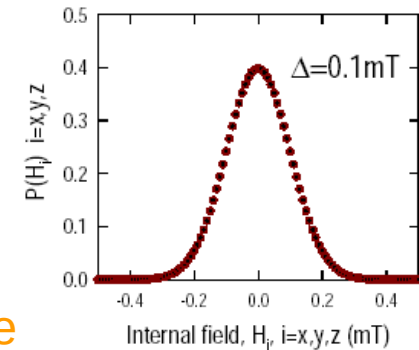
$$\Delta^2 / \gamma_\mu^2 = \langle B^2 \rangle \quad \text{2nd moment field distribution}$$

- Lorentzian field distribution, width Δ
example: dilute magnetic alloy

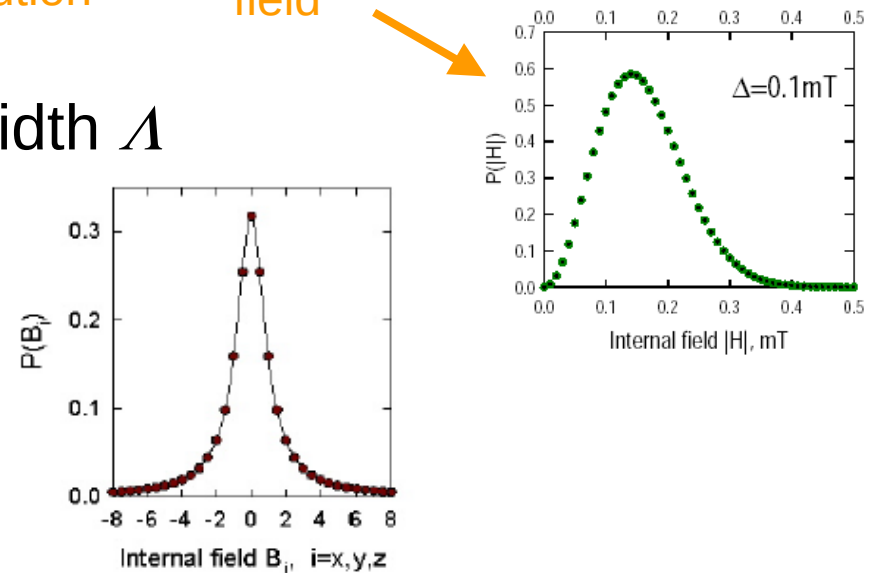
$$f^L(B_i) = \frac{\gamma_\mu}{\pi} \frac{a}{a^2 + \gamma_\mu^2 B_i^2}$$

$$a = \gamma_\mu \Delta$$

components field



magnitude field



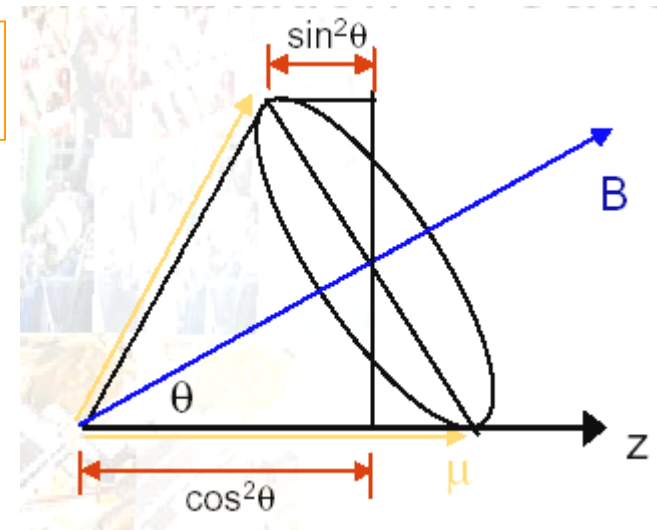
Relaxation function

- z-component of the muon spin

$$s_z(t) = \cos^2 \theta + \sin^2 \theta \cos(\gamma_\mu B t)$$

time independent
component

precessing
component



- relaxation function is given by the statistical average of $s_z(t)$

$$G_z(t) = \iiint s_z(t) f(B_x) f(B_y) f(B_z) dB_x dB_y dB_z$$

Static relaxation, Gauss, ZF

- Larmor equation, δ function distribution, single domain,

$$G(t) = \cos^2 \theta + \sin^2 \theta \cos(\gamma_\mu B t)$$

- polycrystalline average

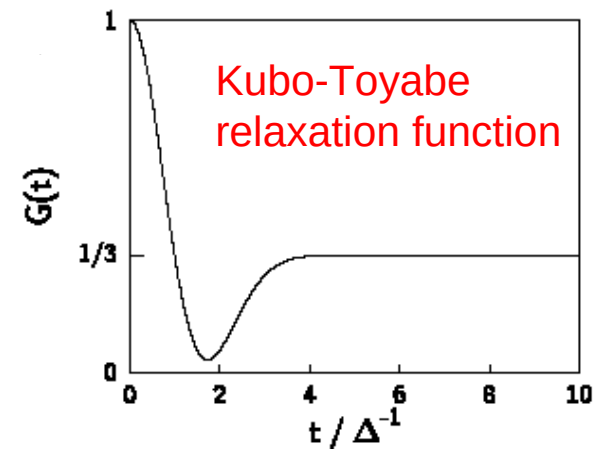
$$G(t) = \frac{1}{3} + \frac{2}{3} \cos(\gamma_\mu B t)$$

- Gaussian distribution of static moments

inhomogeneous line broadening

nuclear moments, weak magnetic order

$$G_{KT}(t) = \frac{1}{3} + \frac{2}{3} (1 - \Delta^2 t^2) \exp\left(-\frac{1}{2} \Delta^2 t^2\right)$$



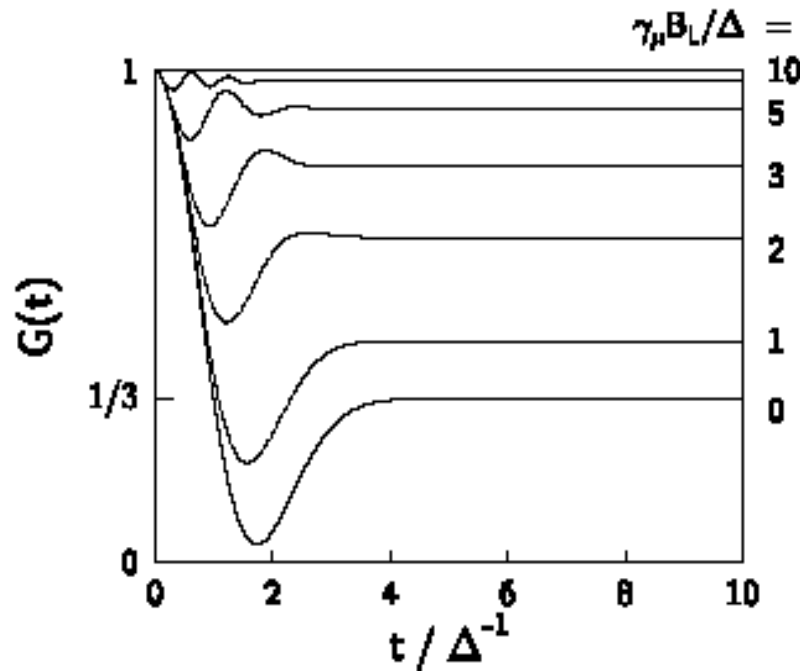
Static relaxation, Gauss, LF

$$G_{KT}(t, B_L) = 1 - \left(\frac{2\Delta^2}{\omega_L^2} \right) \left(1 - e^{-\frac{1}{2}\Delta^2 t^2} \cos(\omega_L t) \right) + \left(\frac{2\Delta^4}{\omega_L^3} \right) \int_0^t e^{-\frac{1}{2}\Delta^2 \tau^2} \sin(\omega_L \tau) d\tau$$

$$\omega_L = \gamma_\mu B_L$$

Gaussian
distribution

“decoupling”
muon spin from
internal fields

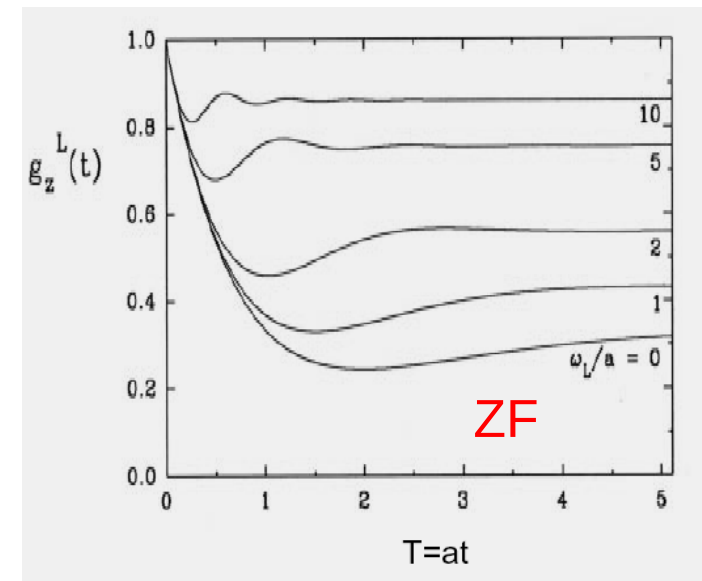


Static relaxation, Lorentzian

- ZF $G_L(t) = \frac{1}{3} - \frac{2}{3}(1 - at) \exp(-at)$ $a = \gamma_\mu \Lambda$

- LF

$$G_L(t, \omega_L) = 1 - \frac{a}{\omega_L} j_1(\omega_L t) e^{-at} - \left(\frac{a}{\omega_L} \right)^2 \left(j_0(\omega_L t) e^{-at} - 1 \right) - \left(1 + \left(\frac{a}{\omega_L} \right)^2 \right) a \int_0^t j_0(\omega_L \tau) e^{-a\tau} d\tau$$



in field

j_0 and j_1 spherical Bessel functions

Dynamic relaxation

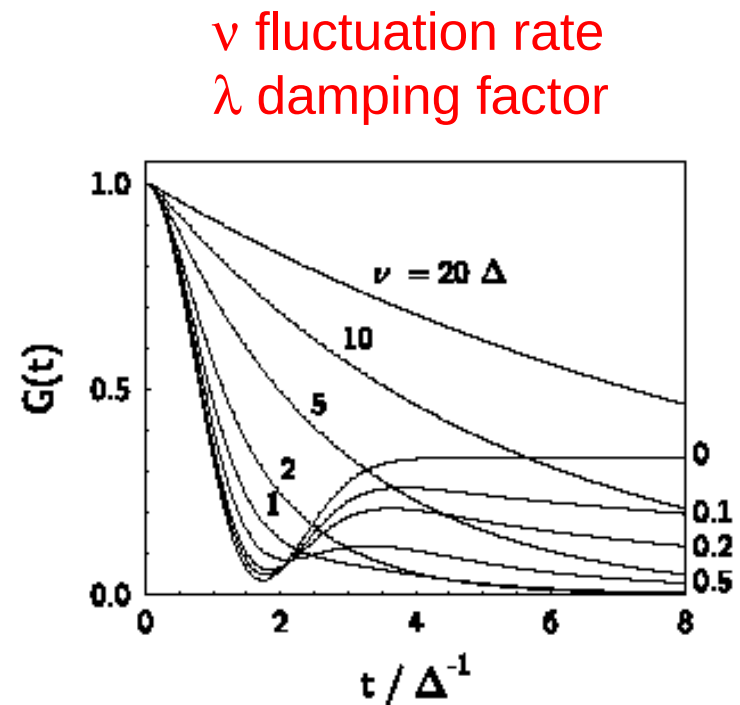
- internal field dynamics
muon hopping, fluctuating fields
- dynamical Kubo-Toyabe function $G_{\text{DKT}}(t)$

- limit Gaussian fast fluctuations, ZF
motional narrowing

$$G(t) = e^{-2\Delta^2 t / \nu} = e^{-\lambda t}$$

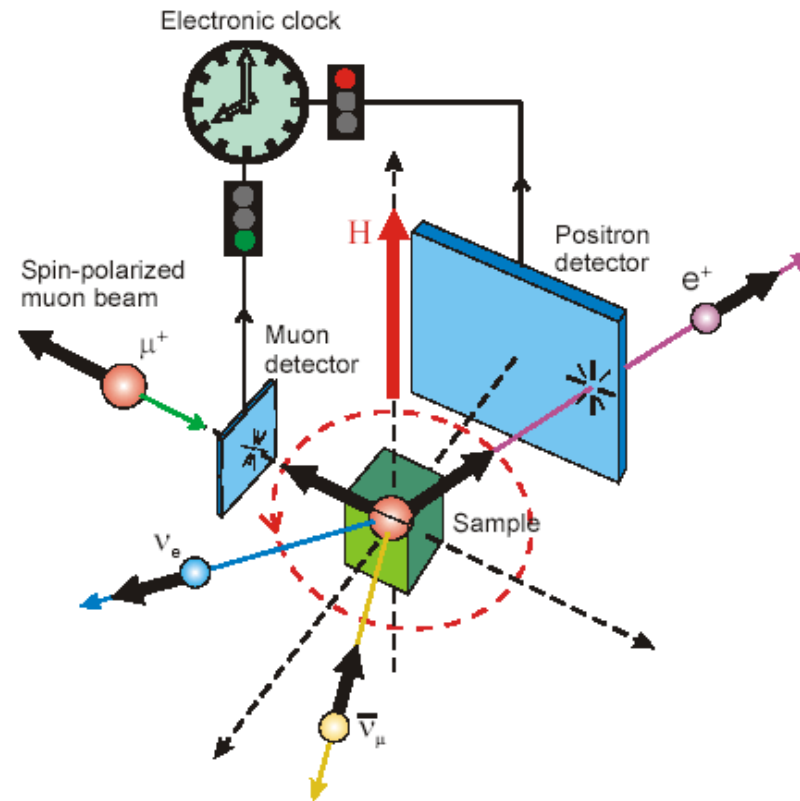
- Gaussian slow fluctuations, ZF
motional averaging

$$G(t, \nu) = \frac{1}{3} e^{-\frac{2}{3}\nu t}$$



3. Muon spin rotation

- Transversal field: $B_{\text{ext}} \perp P_{\mu}(0)$
→ determine muon stopping site



Local field at the muon site

- field at interstitial site

$$\vec{B}_\mu = \vec{B}_{ext} + \vec{B}_{hf} + \vec{B}_{dem} + \vec{B}_L$$

hyperfine
field

$$\vec{B}_{h_i^c} = \vec{B}_{con} + \vec{B}_{dip}$$

$$\vec{B}_{con} = \vec{A}_{con} \vec{\chi}_P \vec{B}_{ext}$$

conduction electron susceptibility

local moment susceptibility

$$\vec{B}_{dip} = \vec{A}_{dip} \vec{\chi}_{CW} \vec{B}_{ext}$$

dipolar field

$$\vec{B}_{dem} = -\vec{N}\vec{M}$$

demagnetization
field

$$\vec{A}_{dip}(i, j) = \sum_{r_f} \left(\frac{1}{r^3} \frac{3r_i r_j}{r^2} - \delta_{ij} \right)$$

dipolar tensor
(moments at lattice sites f)

$$\vec{B}_L = -\frac{1}{3} \mu_0 \vec{M}$$

Lorentz field

$$\vec{r} = \vec{r}_f - \vec{r}_\mu = (r_1, r_2, r_3) = (x, y, z) \quad ; \quad r = |\vec{r}|$$

μ^+ Knight shift

- μ^+ frequency shift

$$K_\mu = \frac{|\vec{B}_\mu| - |\vec{B}_{ext}|}{|\vec{B}_{ext}|} = \frac{\omega_\mu}{\omega_{ext}} - 1 \quad \omega_{ext} = \gamma_\mu B_{ext}$$

- neglect demagnetization + Lorentz

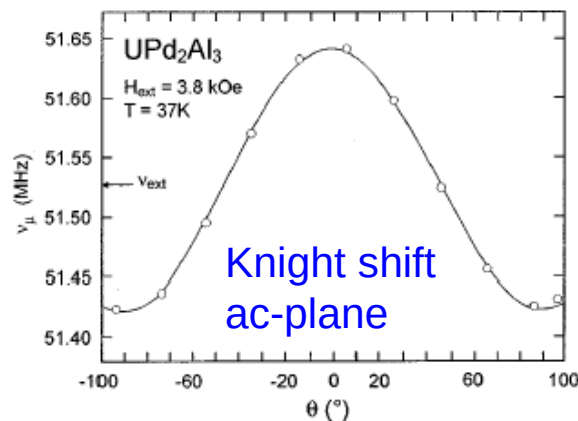
$$K_\mu = K_{con} + \frac{\vec{B}_{ext}}{|\vec{B}_{ext}|} \cdot \left(\vec{A}_{dip} \vec{\chi}_{CW} \frac{\vec{B}_{ext}}{|\vec{B}_{ext}|} \right)$$

$\uparrow$ $\neq f(T)$ $\uparrow$ $= f(T)$

- Knight shift $\propto$ susceptibility

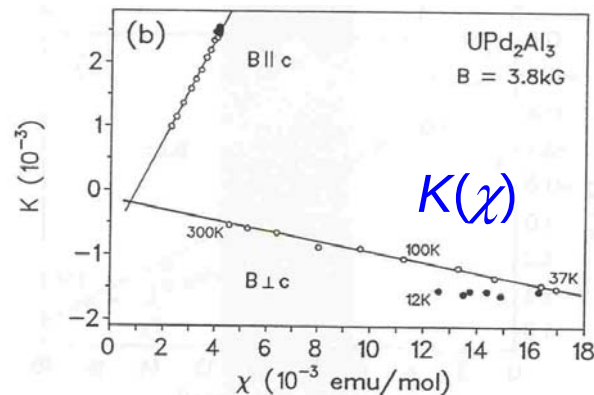
$$K_i = A^{ii} \chi_{ii}$$

Example determination μ^+ stopping site

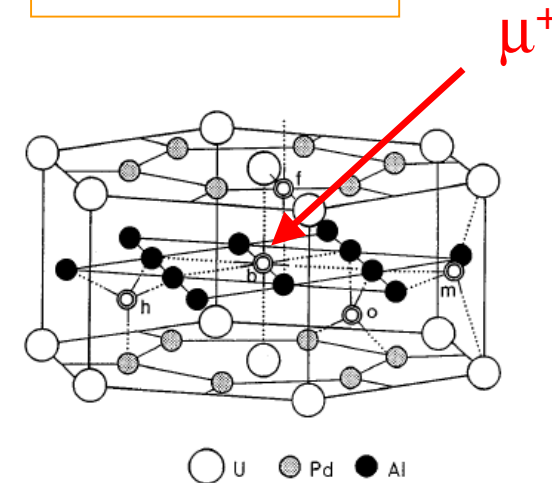


1 frequency
→ 1 stopping site

UPd₂Al₃
hexagonal
structure



$$K_i(\chi_{ij}) \rightarrow A_{ij}$$



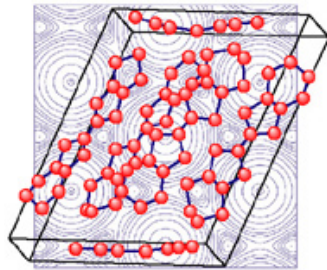
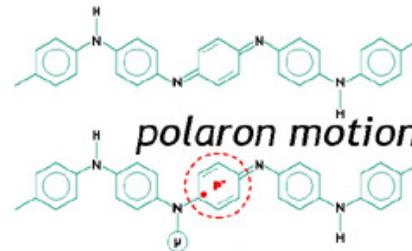
comparison exp. and calculated
dipolar tensor

→ stopping site: $(0,0,\frac{1}{2})$, b-site

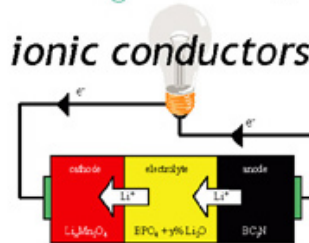
Feyerherm 1995

4. Examples and analysis

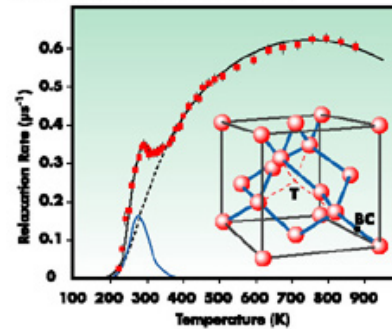
superconductors



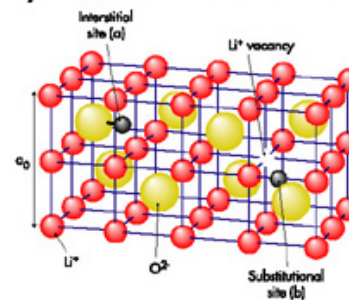
ionic conductors



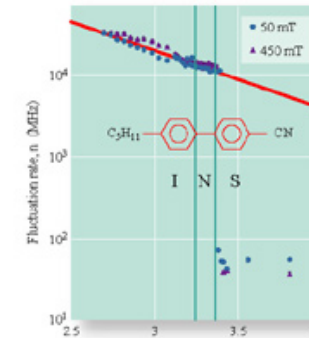
semiconductors



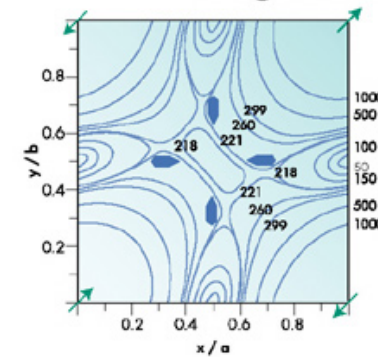
proton conductors



molecular dynamics

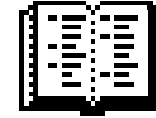


magnetism



Case studies

HOMEWORK Study one of the following papers

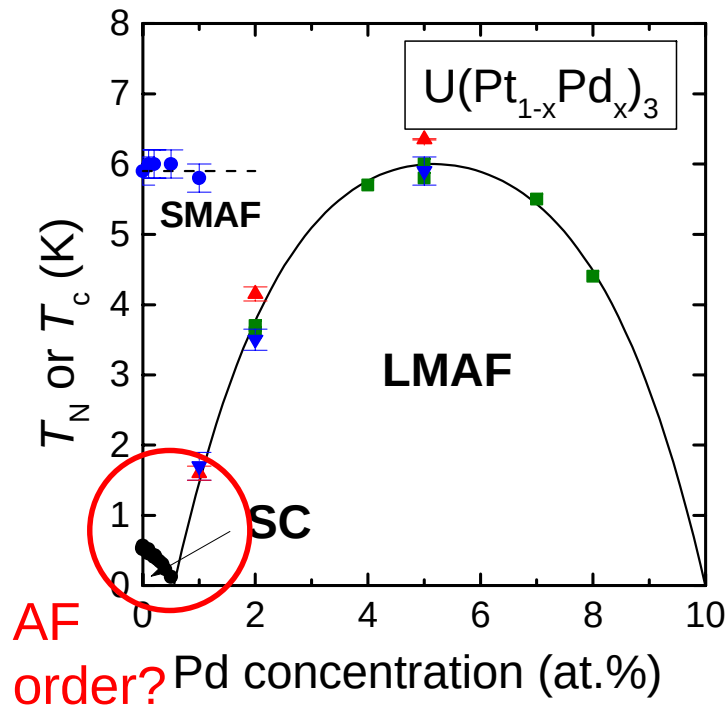


- Time reversal symmetry-breaking superconductivity in Sr_2RuO_4
G.M. Luke *et al.*, Nature 394 (1998) 558.
- Anomalous weak magnetism in superconducting $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$
J.E. Sonnier *et al.*, Science 292 (2001) 1692.
- Muon spin relaxation and isotropic pairing in superconducting $\text{PrOs}_4\text{Sb}_{12}$
D.E. MacLaughlin *et al.*, Phys. Rev. Lett. 89 (2002) 157001.

On the following 3 slides case study “Emerging magnetism in $\text{U}(\text{Pt},\text{Pd})_3$ ”
A. de Visser *et al.*, Phys. Rev. Lett. 85 (2000) 3005.



Emerging magnetism in $\text{U}(\text{Pt},\text{Pd})_3$ slide 1



AdV et al., PRL 85, 3005 (2000)

- TF μSR $B_{\text{ext}} = 0.01$ T
- implanted μ^+ precesses with frequency $\nu_\mu = \gamma_\mu B_{\text{loc}}$
- $B_{\text{dip}} \rightarrow$ dephasing and damping
- T dependence of damping rate $\rightarrow$ information about emerging sources of magnetism
- experiments in LTF $T > 0.05$ K πM3 beam line PSI
- polycrystalline $\text{U}(\text{Pt}_{1-x}\text{Pd}_x)_3$ $0.007 \leq x \leq 0.009$

Emerging magnetism in $\text{U}(\text{Pt},\text{Pd})_3$ slide 2

- μ^+ depolarization function
damped Gauss

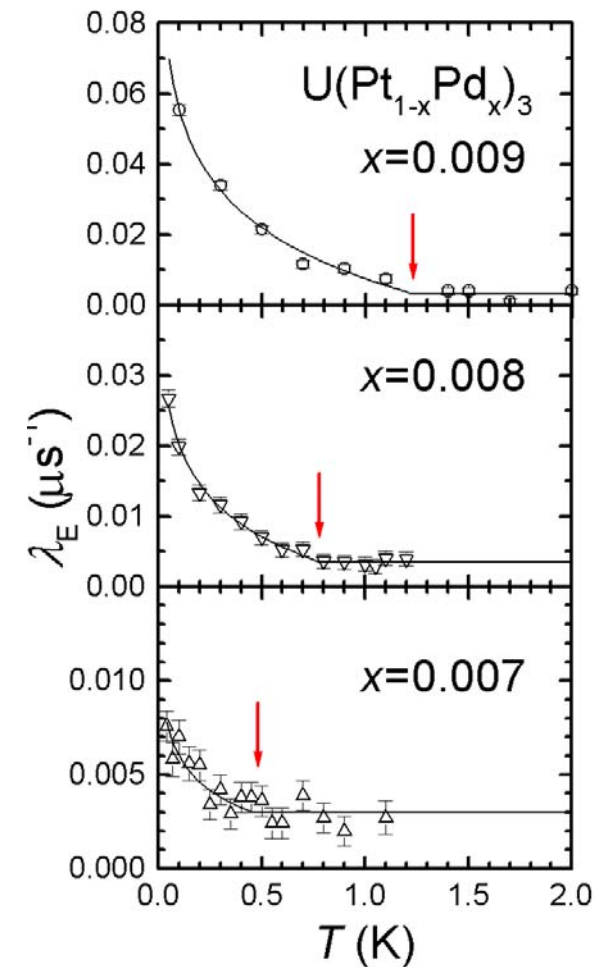
$$P_{DG}(t) = A_{DG} \cos(\omega t) \exp(-\lambda_E t - \Delta^2 t^2 / 2)$$

- $\Delta \approx 0.06 \mu\text{s}^{-1}$ due to
Pt nuclear moments

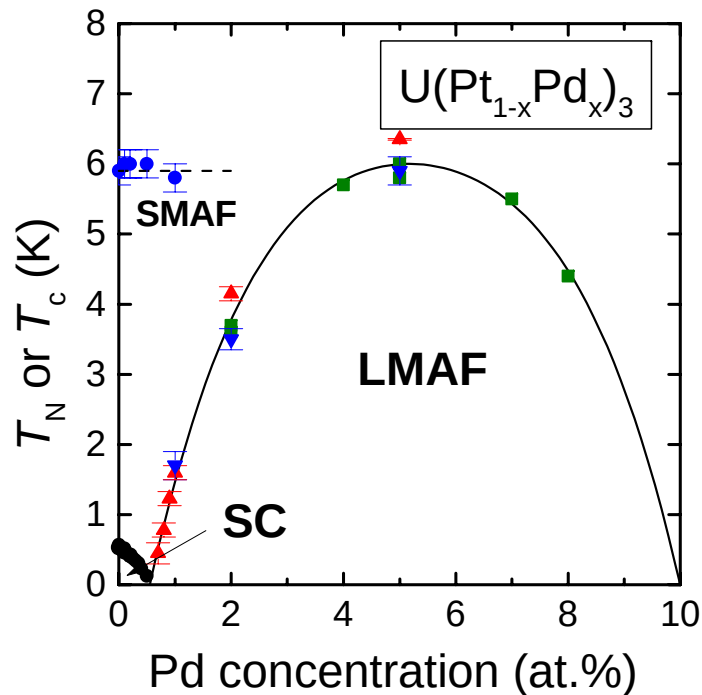
- fit exponential damping

$$\lambda_E = \lambda_{BG} + \lambda_{LMAF}$$

- $\lambda_{LMAF} \sim -\ln(T/T_N)$ for $T < T_N$
- $\lambda_{LMAF} = 0$ for $T > T_N$



Emerging magnetism in $\text{U}(\text{Pt},\text{Pd})_3$ slide 3



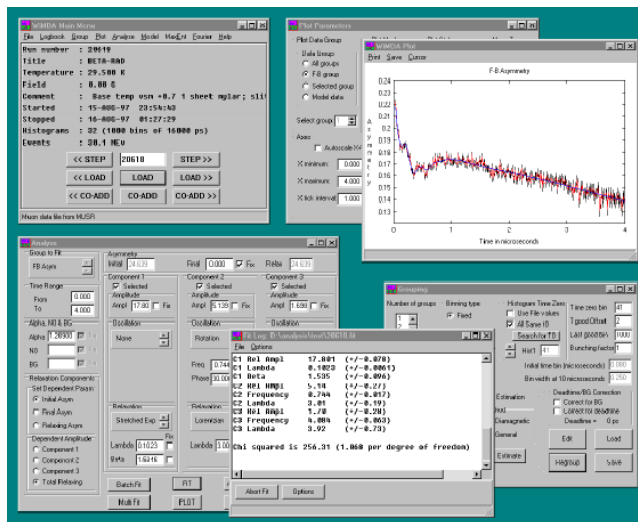
- T_N drops with decreasing x

x	T_N
0.009	1.23 K
0.008	0.78 K
0.007	0.45 K

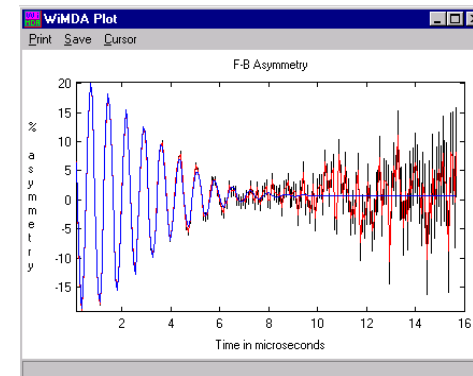
- antiferromagnetic quantum critical point at $x_{c,af} \approx 0.006$

Data analysis

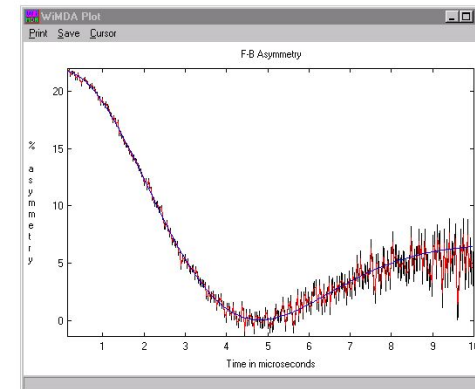
- dedicated programs, e.g.
 - Mufit for PSI data (© Fred Gygax)
 - WiMDA for ISIS data (© Francis Pratt)



screen shot WiMDA



oscillation with Lorentzian decay



Kubo-Toyabe decay

Final recap

- implanted muon precesses in local field
decay positrons emitted
in preferred direction (asymmetry)
- extremely sensitive probe for measuring
internal fields $B_{\text{loc}} \sim 0.1$ Oe
- local probe $\rightarrow$ determine volume fractions
- physics: μ^+ depolarization function $G(t) = P_{\mu}(t)/P_{\mu}(0)$
- static and dynamical processes
- broad spectrum of fluctuations $10^4 - 10^{11}$ Hz

